Principles of AI Planning

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Winter Term 2011/2012

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Exercise Sheet 5

Due: December 6th, 2011

Exercise 5.1 (h^+ heuristic, 2+2 points)

A 15-puzzle planning task $\Pi = \langle A, I, O, \gamma \rangle$ is given as

$$\begin{aligned}
A &= \{empty(p_{i,j}) \mid 0 \leq i,j \leq 3\} \cup \{at(t_k, p_{i,j}) \mid 0 \leq i,j \leq 3, 0 \leq k \leq 14\},\\
O &= \{move(t_m, p_{i,j}, p_{k,l}) \mid 0 \leq i,j,k,l \leq 3, 0 \leq m \leq 14,\\
&\quad (i = k \text{ and } |j - l| = 1) \text{ or } (j = l \text{ and } |i - k| = 1)\},\\
\gamma &= \bigwedge_{0 \leq m \leq 14} at(t_m, p_{\lfloor m/4 \rfloor, m\%4})
\end{aligned}$$

Action $move(t_m, p_{i,j}, p_{k,l})$ moves tile t_m from position $p_{i,j}$ to position $p_{k,l}$:

$$\begin{aligned}
move(t_m, p_{i,j}, p_{k,l}) = \langle & at(t_m, p_{i,j}) \wedge empty(p_{k,l}),\\
& at(t_m, p_{k,l}) \wedge empty(p_{i,j}) \wedge \neg at(t_m, p_{i,j}) \wedge \neg empty(p_{k,l}) \rangle
\end{aligned}$$

A syntactically possible state is *legal* if each tile t_m is at some position p_{ij}, if no two tiles are at the same position and if the remaining position is the only one that is *empty*. The initial state is an arbitrary state that is legal.

One possible heuristic for the 15-puzzle is the Manhattan-distance heuristic $h^{Manhattan}$: It sums the Manhattan distances of all tiles from their current positions to their target positions, where the Manhattan distance between position $p_{i,j}$ and $p_{k,l}$ is given as $|i - k| + |j - l|$.
The h^+ heuristic estimates the distance of state s to the closest goal state as the length of the optimal plan in the relaxed planning task (with initial state s).

(a) Show that $h^+(s) \geq h^{Manhattan}(s)$ for each legal state s of a 15-puzzle planning task.

(b) Show that $h^+(s) > h^{Manhattan}(s)$ for at least one state s of a 15-puzzle planning task.

Exercise 5.2 (Relaxed planning graph and heuristics, 1+3+1+1 points)

Consider the relaxed planning task Π^+ with variables $A = \{a, b, c, d, e\}$, operators $O = \{o_1, o_2\}$, $o_1 = \langle a \vee b, c \wedge d \wedge (c \rhd b) \rangle$, $o_2 = \langle d, e \rangle$, goal $\gamma = b \wedge e$ and initial state $s = \{a \mapsto 1, b \mapsto 0, c \mapsto 0, d \mapsto 0, e \mapsto 0\}$.

(a) Calculate $h^+(s)$ for Π^+ and explain your answer.

(b) Draw the relaxed planning graph $\text{RPG}_k(\Pi^+)$ for depth $k = 2$ and calculate the truth values of the nodes.

(c) Calculate $h_{\max}$ for Π^+ and explain your answer.

(d) Calculate h_{add} for Π^+ and explain your answer.

Hint: If you annotate the relaxed planning graph with numbers representing heuristic values (or truth values) to explain an answer, please indicate clearly the meaning of each number.

Note: The exercise sheets may and should be worked on in groups of two students. Please state both names on your solution (this also holds for submissions by e-mail).