

Theoretical Computer Science II

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Extra Exercise Sheet 3

Exercise 3.1 (Context Free Grammars)

Design context-free grammars for the following languages.

- The set of odd-length strings whose first, middle, and last characters are all the same, over the alphabet $\{0, 1\}$.
- The set of all strings of a 's and b 's that are *not* of the form ww , that is, not equal to any string repeated.
- The set of all strings with an equal number of a 's and b 's.
- The set of all strings with twice as many a 's as b 's.
- Let $T = \{0, 1, (,), +, *, \emptyset, e\}$. You may think of T as the set of the symbols used by regular expressions over the alphabet $\{0, 1\}$; the only difference is that we use e for ϵ to avoid potential confusion in what follows. Design a grammar that generates exactly the regular expressions with alphabet $\{0, 1\}$

Exercise 3.2 (Pushdown Automata)

Construct PDAs for the following languages over $\Sigma = \{0, 1\}$, unless stated otherwise.

- $L_a = \{x \mid x \text{ has the same number of } 0\text{'s and } 1\text{'s, and every prefix has at least as many } 0\text{'s as } 1\text{'s}\}$
- $L_b = \{x \mid x \neq x^R \text{ (} x \text{ is not a palindrome)}.\}$
- $L_c = \{0^n x \mid |x| \leq n\}$.
- $L_d = \{0^m 1^n 2^p \mid m \neq n + p\}$ with $\Sigma = \{0, 1, 2\}$.
- $L_3 = \{0^n 1^m \mid n \leq m \leq 2n\}$

Exercise 3.3 (Chomsky Normal Form)

Transform the following grammars in Chomsky Normal Form

a)

$$\begin{aligned} S &\rightarrow AB \mid CA \\ A &\rightarrow a \\ B &\rightarrow BC \mid AB \\ C &\rightarrow aB \mid b \end{aligned}$$

b)

$$\begin{aligned} S &\rightarrow ASB \mid \epsilon \\ A &\rightarrow aAS \mid a \\ B &\rightarrow SbS \mid A \mid BB \end{aligned}$$

c)

$$\begin{aligned} S &\rightarrow AAA \mid B \\ A &\rightarrow aA \mid B \\ B &\rightarrow \epsilon \end{aligned}$$

Exercise 3.4 (Pumping Lemma for CFL)

Use the CFL pumping lemma to show each of these languages not to be context free

- a) $L_a = \{0^i 1^j 2^i 3^j \mid i \geq 1 \text{ and } j \geq 1\}$.
- b) $L_b = \{ww \mid w \in \{0, 1\}^*\}$.
- c) $L_c = \{a^i b^j c^k \mid i < j < k\}$.
- d) $L_d = \{a^n b^n c^i \mid n \leq i \leq 2n\}$.
- e) $L_e = \{ww^R w \mid w \in \{0, 1\}^*\}$. That is, the set of strings consisting of some string w followed by the same string in reverse, and then the string w again, such as 001100001.