Theoretical Computer Science II
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Extra Exercise Sheet 2

Exercise 2.1 (Deterministic Finite Automata)

Construct DFAs that recognize the following languages. In all cases, $\Sigma = \{0, 1\}$.

- The empty language $\emptyset$.
- Only ϵ.
- All the languages.
- The set of strings with three consecutive 0's (not necessarily at the end).
- The set of strings that does not contain 00.
- The set of strings that either begin or end with the same number twice.
- The set of strings such that each block of five consecutive symbols contains at least two 1's.
- The set of strings such that the number of 0's is divisible by 5, and the number of 1's is divisible by 3.
- The set of strings beginning with a 1 that, when interpreted as a binary integer, is a multiple of 5. For example, strings $101, 1010, 1111$ are in the language and $0, 100, 111$ are not.

Exercise 2.2 (Non Deterministic Finite Automata)

Construct NFAs that recognize the following languages.

- The set of strings where letters appear in reversed alphabetical order. $\Sigma = \{a, b, c\}$.
- The set of strings that contains an a in the odd positions. $\Sigma = \{a, b, c\}$.
- The set of strings that contains at least two occurrences of cb and ends in bb. $\Sigma = \{a, b, c\}$
- The set of strings where the 5th symbol from the end is an a. $\Sigma = \{a, b\}$
- The set of strings over $\Sigma = \{0, 1, \ldots 9\}$ such that the final digit has appeared before.
- The set of strings over $\Sigma = \{0, 1, \ldots 9\}$ such that the final digit has not appeared before.
- The set of strings such that there are no two a's separated by a number of positions that is a multiple of 4. Note that 0 is an allowable multiple of 4.

Exercise 2.3 (Regular Expressions)

Write regular expressions for the following languages:

- The set of strings over alphabet $\Sigma = \{a, b, c\}$ containing at least one a and at least one b.
- The set of strings over alphabet $\Sigma = \{a, b, c\}$ where every a is followed immediately by a c.
- The set of strings over alphabet $\Sigma = \{a, b, c\}$ that have an even number of the substring ac.
- The set of strings of 0's and 1's whose fifth symbol from the right end is a 1.
- The set of strings of 0's and 1's with at most one pair of consecutive 1's.
- The set of strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.
- The set of strings of 0's and 1's whose number of 0's is divisible by five.
- The set of strings of 0's and 1's not containing 101 as a substring.
- The phone numbers in Germany.

Exercise 2.4 (Regular Expressions Properties)

Prove or disprove each of the following statements about regular expressions

- $r + s = s + r$
- $(r^*)^* = r^*$
- $(r + s)^* = r^* + s^*$
- $(rs + r)^* rs = (rr^* s)^*$

Exercise 2.5 (The Pumping Lemma)

Prove that the following are not regular languages by using the pumping lemma.

- $L = \{$All strings with an equal number of $0s$ and $1s$ not in any particular order.$\}$
- $L = \{0^n 110^n \mid n \geq 1\}$
- $L = \{0^n 1^m 0^n \mid n, m \in \mathbb{N}\}$
- $L = \{0^n 1^m \mid n, m \in \mathbb{N}$ such that $n \leq m\}$
- $L = \{0^n 1^{2n} \mid n \geq 1\}$
- $L = \{0^n \mid n$ is a perfect cube$\}$
- $L = \{0^n \mid n$ is a power of $2\}$
- The language of palindromes.
- The set of strings of 0's and 1's that are of the form ww, that is, some string repeated.
- The set of strings of 0's and 1's that are of the form ww^R, that is, some string followed by its reverse.
- The set of strings of 0's and 1's of the form $w\overline{w}$, where $\overline{w}$ is formed from w by replacing all 0's by 1's and vice versa; e.g., $\overline{011} = 100$ and 011100 is an example of a string in the language.