

Theoretical Computer Science II

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Winter semester 2011/2012

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Extra Exercise Sheet 1

Exercise 1.1 (Proof by Induction)

Prove the following statements by Induction. Make clear what is the *Base Case*, the *Induction Hypothesis* and the *Induction Step*.

- a) For any $n \geq 1$

$$1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$$

- b) If every two cities in state A are joined by a one-way road, then it is possible to find a starting city in A and a route across A that passes through every city exactly once.
- c) For some $h \in \mathbb{N}$ show that the following characterization of Fibonacci numbers holds:

$$F_h = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^{h+1} - \left(\frac{1 - \sqrt{5}}{2} \right)^{h+1} \right)$$

Exercise 1.2 (Propositional Logic)

Translate the following sentences into Propositional Logic.

- a) It is false that both the alarm is switched on and it is switched off.
- b) Alarm is switched on if and only if the door is open and no card inserted. Therefore, if the door is closed then the alarm is switched off.

Is this argument satisfiable? Is it valid?

Exercise 1.3 (Resolution)

Translate the following sentences in Propositional Logic. Then use resolution on the given hypotheses to derive the conclusion.

- a) If it rains, Joe brings his umbrella. If Joe has an umbrella, he doesn't get wet. If it doesn't rain, Joe doesn't get wet.

Prove that Joe doesn't get wet.

- b) Either Heather attended the meeting or Heather was not invited. If the boss wanted Heather at the meeting, then she was invited. Heather did not attend the meeting. If the boss did not want Heather there, and the boss did not invite her there, then she is going to be fired.

Prove Heather is going to be fired.

- c) Either taxes are increased or if expenditures rise then the debt ceiling is raised. If taxes are increased, then the cost of collecting taxes rises. If a rise in expenditures implies that the government borrows more money, then if the debt ceiling is raised, then interest rates increase. If taxes are not increased and the cost of collecting taxes does not increase then if the debt ceiling is

raised, then the government borrows more money. The cost of collecting taxes does not increase. Either interest rates do not increase or the government does not borrow more money.

Prove that either the debt ceiling isn't raised or expenditures don't rise.

Exercise 1.4 (Formalization in Predicate Logic)

Represent the following sentences in Predicate Logic. First create a consistent vocabulary, e.g., as the (uncomplete) example shown below.

$Takes(x, c, s)$	student x takes course c in semester s
$Shaves(x, y)$	person x shaves person y
$Fools(x, y, t)$	person x fools person y in time t
...	...
	Student(x), Person(x), ...

1. All basketballs are made of rubber.
2. Every flower located in Sue's yard is red.
3. Some students took English in spring 2010.
4. Every student who takes English passes it.
5. Exactly one exercise in Computer Theory II was hard.
6. There is a barber who shaves all men in town who do not shave themselves.
7. Politicians can fool some of the people all the time, and they can fool all the people some of the time, but they cannot fool all the people all of the time.
8. All birds can fly except for penguins and ostriches and those with a broken wing.

Exercise 1.5 (Formalization in Predicate Logic)

Write a general set of facts and axioms to represent the sentence "Brutus heard about Julius Caesar's death" and to correctly answer the question "Did Julius Caesar hear about Brutus' death?"