Theoretical Computer Science II

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Exercise Sheet 14

Due: February 13, 2012

Exercise 14.1 (P, 1.5 + 0.5 marks)

(a) Show that P is closed under union, complement, and concatenation.

(b) The complexity class coP contains all languages L whose complement is in P. Formally, $coP = \{L \mid \bar{L} \in P\}$. Is $P = coP$?

Exercise 14.2 (Post's Correspondence Problem, 2 + 2 marks)

An instance of *Post's Correspondence Problem* (PCP) consists of two lists of strings over some alphabet Σ; the two lists must be of equal length, and we generally refer to them as A and B. We write $A = w_1, w_2, \ldots, w_k$ and $B = x_1, x_2, \ldots, x_k$, for some integer k. For each i, the pair (w_i, x_i) is said to be a *corresponding* pair.

We say this instance of PCP *has a solution*, if there is a sequence of one or more integers $i_1, i_2, \ldots, i_m$ that, when interpreted as indexes for strings in the A and B lists, yield the same string. That is, $w_{i_1} w_{i_2} \ldots w_{i_m} = x_{i_1} x_{i_2} \ldots x_{i_m}$. We say that the sequence $i_1, i_2, \ldots, i_m$ is a solution to this instance of PCP.

Consider the following instances Y of PCP. Which instance Y has a solution? Justify your answer.

a) $Y = \{(aba, a), (ba, babab)\}$

b) $Y = \{(bab, ba), (aaabb, a), (ab, abbab)\}$

Exercise 14.3 (Reduction, 1 + 3 marks)

A *k-clique* in a graph G is a set of k nodes of G such that there is an edge between every two nodes in the clique. Thus, a $2-clique$ is just a pair of nodes connected by an edge, and a $3-clique$ is a triangle. Consider the following problems.

$$CLIQUE = \{\langle G, k\rangle \mid G \text{ is an undirected graph that contains a clique with } k \text{ vertices}\}$$
$$NODE\ COVER = \{\langle G, k\rangle \mid G \text{ is an undirected graph that has a node cover with } k \text{ or fewer nodes.}\}$$

(a) How many edges does a $k-clique$ have, as a function of k?

(b) Prove that CLIQUE is $NP-$complete by reducing the node-cover problem to CLIQUE.