

## Theoretical Computer Science II

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Winter semester 2011/2012

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### Exercise Sheet 12

**Due: January 30, 2012**

**Exercise 12.1** (Decidable Languages, 1 + 1 marks)

Let  $L$  and  $L'$  be decidable languages. Prove the following properties.

- (a) The complement  $\bar{L}$  is decidable.
- (b) The union  $L \cup L'$  is decidable.

**Exercise 12.2** (Decidable Languages, 1.5 + 1 + 1.5 marks)

Show that the following languages are decidable:

- (a)  $EQ_{DFA\_RE} = \{\langle D, R \rangle \mid D \text{ is a DFA and } R \text{ is a regular expression and } L(D) = L(R)\}$
- (b)  $A_{\epsilon CFG} = \{\langle G \rangle \mid G \text{ is a CFG that generates } \epsilon\}$
- (c)  $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA that recognizes } \Sigma^*\}$

**Exercise 12.3** (Undecidable Languages, 4 marks)

Consider the problem of determining whether a two-tape Turing machine ever writes a non-blank symbol on its second tape, i.e.

$$N = \{\langle M, w \rangle \mid M \text{ is a two-tape Turing machine which writes a non-blank symbol onto its second tape when it runs on } w\}.$$

Show that  $N$  is undecidable. *Hint:* Use a reduction from  $A_{TM}$ .