

Theoretical Computer Science II

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Exercise Sheet 5

Due: November 28, 2011

Exercise 5.1 (Predicate Logic – Free and bound variables, 2 marks)

Mark the free occurrences of variables in the following formulae and specify the set of free variables for each of these formulae. (In the underlying signature, x, y and z are variable symbols and a and b are constant symbols.)

- (a) $P(a, y)$
- (b) $P(x, y) \wedge \exists x \forall z (Q(x, y, z))$
- (c) $\exists x (P(x, y) \wedge Q(x)) \vee P(y, x)$
- (d) $\forall x (\exists y (P(x, y) \wedge Q(x)) \vee P(x, y))$
- (e) $\forall x \forall y (P(x, b) \wedge Q(x) \vee P(f(y), x))$
- (f) $\exists x (\exists z (R(x, z)) \vee \forall y (S(y, g(x, z))))$
- (g) $\exists x (\exists z (R(x, z))) \vee \forall y (S(y, g(x, z)))$

Exercise 5.2 (Entailment in Predicate Logic, 3 marks)

Prove that $\forall x \exists y P(x, y) \models \exists x P(f(z), x)$.

Exercise 5.3 (Equivalences in Predicate Logic, 2.5 + 2.5 marks)

- (a) Prove the equivalence $\neg \forall x \phi \equiv \exists x \neg \phi$.
- (b) Show that in general it does *not* hold that $\forall x \exists y \phi \equiv \exists y \forall x \phi$ by giving a counter example.