Theoretical Computer Science II

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Winter semester 2011/2012

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Exercise Sheet 4

Due: November 21, 2011

Exercise 4.1 (Predicate Logic – Terminology, 2 marks)

Classify the following expressions as *terms*, *ground terms*, *atoms*, *formulae*, or *meta language* (statements that are not part of predicate logic itself but statements about the semantics). If there is more than one possibility for an expression, please list them all. In the expressions, a and b are constant symbols, x and y are variable symbols, f and g are function symbols, and P and Q are relation symbols.

(a) $\mathrm{P}(x, y)$

(b) $\mathrm{f}(\mathrm{a}, \mathrm{b})$

(c) $\mathcal{I} \models \mathrm{P}(\mathrm{a}, \mathrm{f}(\mathrm{b}))$

(d) $\mathcal{I}, \alpha \models \mathrm{P}(\mathrm{a}, \mathrm{f}(x))$

(e) $\mathrm{f}(\mathrm{g}(x), \mathrm{b})$

(f) $\mathrm{Q}(x)$ is satisfiable.

(g) $\exists x(\mathrm{P}(x, y) \wedge \mathrm{Q}(x)) \vee \mathrm{P}(y, x)$

(h) $\forall x(\exists y(\mathrm{P}(x, y) \wedge \mathrm{Q}(x)) \vee \mathrm{P}(x, y))$

(i) $\forall x \forall y(\mathrm{P}(x, y) \wedge \mathrm{Q}(x) \vee \mathrm{P}(\mathrm{f}(y), x))$

(j) $\mathrm{Q}(x) \vee \mathrm{P}(x, y) \equiv \mathrm{P}(x, y) \vee \mathrm{Q}(x)$

Exercise 4.2 (Predicate Logic – Interpretations, 4+1 marks)

Consider the following set of formulae:

$$KB = \left\{ \begin{array}{l} \forall x \neg \mathrm{P}(x, x) \\ \forall x \forall y \forall z((\mathrm{P}(x, y) \wedge \mathrm{P}(y, z)) \rightarrow \mathrm{P}(x, z)) \\ \forall x \forall y(\mathrm{P}(x, y) \vee x = y \vee \mathrm{P}(y, x)) \end{array} \right\}$$

(a) Specify an interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{D} = \{d_1, \ldots, d_4\}$ and prove that $\mathcal{I} \models KB$ (i.e., $\mathcal{I} \models \varphi$ for all $\varphi \in KB$). Why is it not necessary to specify a variable assignment α to state a model of KB?

(b) Are there also models of KB with an infinite domain $\mathcal{D}$? If yes, give such an interpretation. If not, justify your answer.

Exercise 4.3 (Logic Applet, 3 marks)

Consider the Logic Applet at `http://www.cs.plattsburgh.edu/~salvador/Tarski/index.html`. To play around with it, please note a description of the syntax and predicates used, as well as some examples.

Use this logic applet to make an arrangement of geometrical figures such that all the expressions given below are true. A maximum of six objects is allowed.
(Note: x, y, and z are variables, whereas e, f, l, m, r, and p are constants, which are used to name objects. Please turn the page..)

(a) $\exists x \exists y \exists z (\text{Pentagon}(x) \wedge \text{Triangle}(y) \wedge \text{Square}(z) \wedge \text{Between}(x, y, z))$

(b) $\forall x \forall y ((\text{Square}(x) \wedge (\text{Square}(y)) \Rightarrow \text{SameRow}(x, y))$

(c) $\forall x \forall y (\text{Triangle}(x) \wedge \text{Square}(y) \Rightarrow \neg\text{SameRow}(x, y))$

(d) $\text{LeftOf}(m, r)$

(e) $\text{LeftOf}(l, p) \wedge \text{LeftOf}(p, r)$

(f) $\forall x \forall y (\text{Triangle}(x) \wedge \text{Triangle}(y) \wedge \text{LeftOf}(x, y) \Rightarrow \text{Smaller}(x, y))$

(g) $\text{Square}(l) \wedge \text{Small}(l) \wedge \text{LeftOf}(l, r)$

(h) $\forall x \exists y (\text{Pentagon}(x) \wedge \text{SameCol}(x, y) \Rightarrow \text{Large}(y))$

(i) $\text{SameCol}(f, r) \wedge \text{SameCol}(e, l)$

(j) $\text{Triangle}(e) \wedge \text{Triangle}(f) \wedge \text{Square}(m) \wedge \text{Square}(r) \wedge \text{Pentagon}(p)$