Theoretical Computer Science II

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Exercise Sheet 1

Due: October 31, 2011

Exercise 1.1 (Proof by induction, 3 marks)

Prove the following statement *by induction*. Please make clear what is the *base case*, the *induction hypothesis* and the *induction step*.
For $n \in \mathbb{N}^+$, the power set of the set $S_n = \{1, 2, \ldots, n\}$ contains 2^n elements.
Explanation: The power set $\mathcal{P}(S)$ of a set S is the set of all subsets of S, e.g. $\mathcal{P}(\{1,2\}) = \{\{\}, \{1\}, \{2\}, \{1,2\}\}$.

Exercise 1.2 (What is wrong: Proof by induction, 2 marks)

Identify the wrong step in the following proof and explain why it is wrong.

The set of animals A is partioned into the non-empty sets of male animals M and female animals F, i.e., for each animal $e \in A$, either $e \in F$ and $e \notin M$ or $e \in M$ and $e \notin F$.

Claim: Each subset of A contains either exclusively elements from F or from M.

Basis: For all subsets of size 1, the claim is true.

Induction hypothesis: For all $k < n$, the claim is true.

Induction step: Consider a subset of $S \subseteq A$ with $n + 1$ elements. Remove an arbitrary element e from S resulting in S'. Since S' has only n elements, the induction hypothesis applies and either $S' \subseteq F$ or $S' \subseteq M$. Now remove a different element e' from S resulting in S''. Again, because of the induction hypothesis, $S'' \subseteq M$ or $S'' \subseteq F$. So, it must be the case that the removed elements have always the same gender as the remaining set. So, also for the set S it holds that $S \subseteq F$ or $S \subseteq M$, which completes the proof.

Exercise 1.3 (Direct proof, 2 marks)

Proof that if a natural number n is uneven, so is n^2.

Exercise 1.4 (Indirect proof, 3 marks)

Proof that there are no two positive integers x, y such that $x^2 - y^2 = 10$.