Constraint Satisfaction Problems

Greedy Local Search

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based on a slideset by

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Greedy Local Search

Constraint solving techniques so far discussed:

- Inference
- Search
- Combinations of inference and search
  $\rightsquigarrow$ improve overall performance; nevertheless worst-time complexity is high

$\Rightarrow$ approximate solutions, for example, by greedy local search methods

$\Rightarrow$ in particular of interest, when we look at optimization problems (e.g. traveling salesman problem, minimize violations of so-called soft constraints)

Stochastic Greedy Local Search (SLS)

Features:

- greedy, hill-climbing traversal of the search space
- in particular, no guarantee to find a solution even if there is one
- search space: states correspond to complete assignment of values to all variables of the constraint network, which are not necessarily solutions of the network
- no systematic search

The SLS-Algorithm

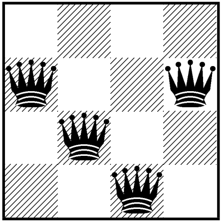
SLS $(\mathcal{C}, \textbf{max\_tries}, \textbf{cost})$:

Input: a constraint network $\mathcal{C}$, a number of tries max_tries, a cost function cost

Output: A solution of $\mathcal{C}$ or "false"

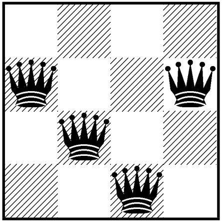
repeat max_tries times
    instantiate a complete random assignment $\overline{a} = (a_1, \ldots, a_n)$
    repeat
        if $\overline{a}$ is consistent **then return** $\overline{a}$
        else let Y be the set of assignments that differ from $\overline{a}$ in exactly one variable-value pair (i.e., change one v_i value a_i to a new value a_i')
            $\overline{a} \leftarrow$ choose an $\overline{a}'$ from Y with maximal cost improvement
        endif
    until current assignment cannot be improved
endrepeat
return "false"

Example



$c(a) = 4$

Example



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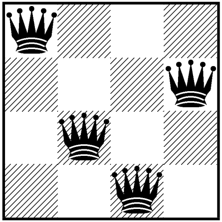

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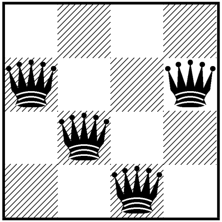

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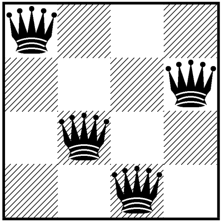

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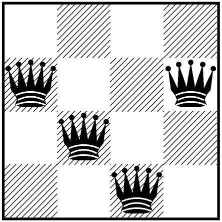

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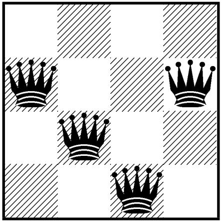

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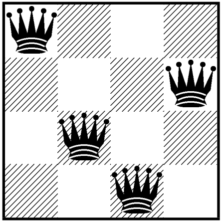

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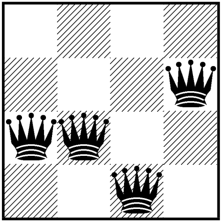
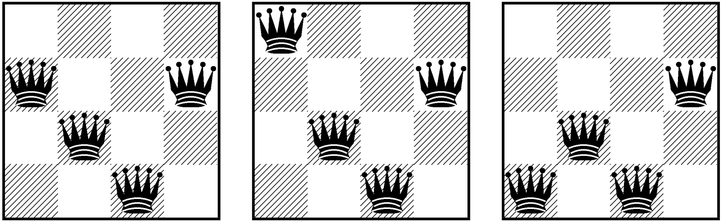


$c(a) = 2$

Example



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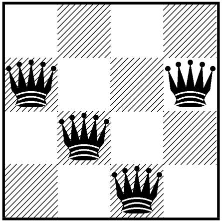

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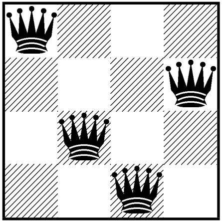

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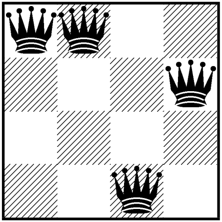

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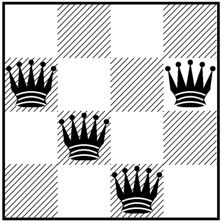

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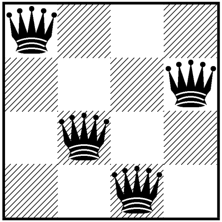

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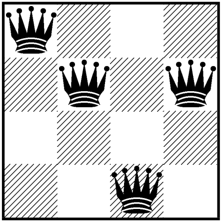

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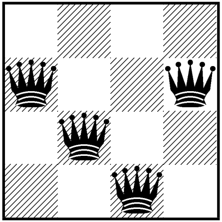

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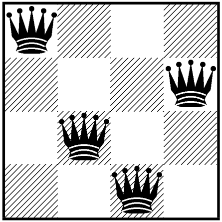

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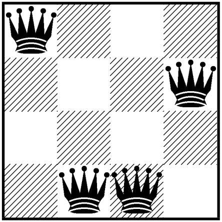

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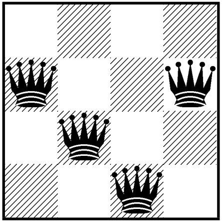

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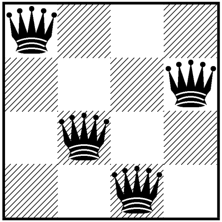

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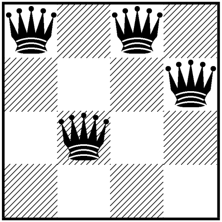

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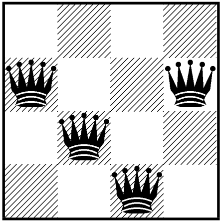

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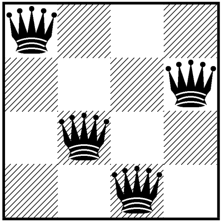

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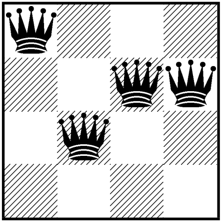

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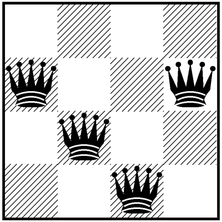

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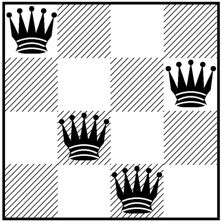

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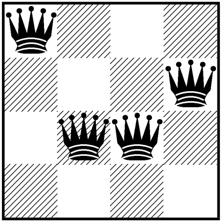

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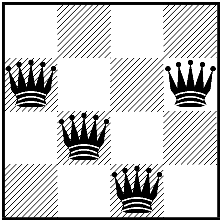
Example



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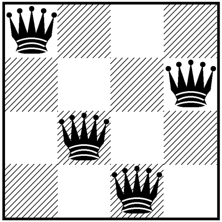

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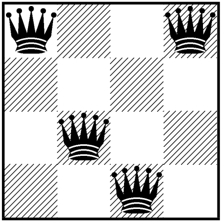

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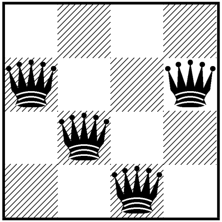

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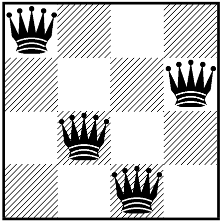

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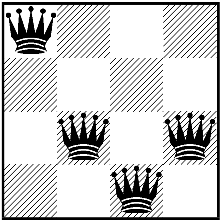

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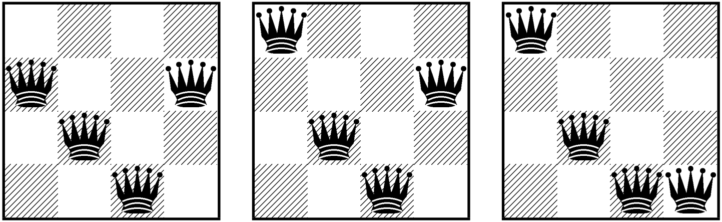

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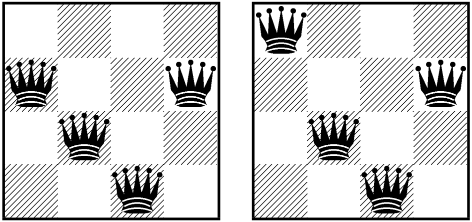

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. . . is a local minimum, from which we cannot escape in SLS

Improvements

In principal, there are two ways for improving the basic SLS-algorithm:

- different strategies for escaping local minima
- other policies for performing local changes

Heuristics for Escaping Local Minima

- Plateau Search: allow for continuing search by sideways moves that do not improve the assignment
- Constraint weighting/ breakout method: as a cost measure use a weighted sum of violated constraints; initial weights are changed when no improving move is available. *Idea:* if no change reduces the cost of the assignment, increase the weight of those constraints that are violated by the current assignment.
- Tabu search: prevent cycling over assignments of the same cost. For this, maintain a list of "forbidden" assignments, called tabu list (usually a list of the last n variable-value assignments). The list is updated whenever the assignment changes. Then changes to variable assignments are only allowed w.r.t. to variable-value pairs not in the tabu list.

Random Walk

Random walk strategy:

- combines random walk search with a greedy approach (bias towards assignments that satisfy more constraints)
- instead of making greedy moves in each step, sometimes perform a random walk step
- for example, start from a random assignment. If the assignment is not a solution, select randomly an unsatisfied constraint and change the value of one of the variables participating in the constraint.

WalkSAT

WalkSAT:

- initially formulated for SAT solving
- turns out to be very successful (in empirical studies)
- based on a two-stage process for selecting variables: in each step select first a constraint violated by the current assignment; second make a random choice between
  a) changing the value of one of the variables in the violated constraint;
  b) minimizing in a greedy way the break value, i.e., the number of new constraints that become inconsistent by changing a value

  The choice between (a) and (b) is controlled by a parameter p (probability for (a))

WalkSAT ($\mathcal{C}$, **max_flips**, **max_tries**):

Input: a constraint network $\mathcal{C}$, numbers max_flips (flips) and max_tries (tries)
Output: "true" and a solution of $\mathcal{C}$, or
"false" and some inconsistent best assignment

$\overline{a}'$ ← a complete random assignment
repeat max_tries times
    $\overline{a}$ ← a complete random assignment
    repeat max_flips times
        if $\overline{a}$ is consistent **then return** "true" and $\overline{a}$
        else select a violated constraint
            with probability p choose an arbitrary variable-value pair (x, a') or,
                with probability $1 - p$, choose a variable-value pair (x, a') that
                minimizes the number of new constraints that break when x's
                value is changed to a' (-1 if the current constraint is satisfied)
            $\overline{a} \leftarrow \overline{a}$ with $x \mapsto a'$
        endif
    endrepeat
    compare $\overline{a}$ with $\overline{a}'$ and retain the better one as $\overline{a}'$
endrepeat
return "false" and $\overline{a}'$

Simulated Annealing

Simulated Annealing:

- *Idea:* over time decrease the probability of doing a random move over one that maximally decreases costs. Metaphorically speaking, by decreasing the probability of random moves, we "freeze" the search space.
- At each step, select a variable-value pair and compute the change of the cost function, δ, when the value of the variable is changed to the selected value. Change the value if δ is not negative (i.e., costs do not increase). Otherwise, we perform the change with probability $e^{-\delta/T}$ where T is the temperature parameter.
- If the temperature T decreases over time, more random moves are allowed at the beginning and less such moves at the end.

Hybrids of Local Search and Inference

SLS-algorithms can also be combined with inference methods. For example, apply SLS only after preprocessing a given CSP instance with some consistency-enforcing algorithm.
Idea: Can we improve SLS by looking at equivalent but more explicit constraint networks?
Note:

- there are classes of problems, e.g., 3SAT problems, which can easily be solved by a systematic backtracking algorithm, but are hard to be solved via SLS
- consistency-enforcing algorithms can change the costs associated to an arc in the constraint graph drastically: assignments near to a solution (in terms of costs) may be very far from a solution after applying inference methods

Example:

- Local search on cycle cutsets

Local Search on Cycle Cutsets

Idea for a hybrid algorithm:

1. Determine a cycle cutset
2. Find some assignment for the cutset variables
3. Propagate values, i.e., find assignment for the tree variables that minimize costs (how do we do that?)
4. Do stochastic local search by varying the cutset variables only
5. Continue with step 3 if there was some improvement
6. Otherwise stop

Usually outperforms pure SLS, provided the cutset is small ($\leq$ 30%).

MinCostTree $(\mathcal{C}, Y, Z, \overline{y})$:

Input: constraint network $\mathcal{C}$, cutset variables Y and tree variables Z with $Y \cup Z = V$ and a partial assignment $\overline{y}$ to the cutset variables
Output: assigment $\overline{z}$ to the variables Z minimizing constraint violations
Comment: $R_{z_i,z_j}(a_i, a_j) = 1$ if $(a_i, a_j) \in R_{z_i,z_j}$, otherwise it is 0.
Compute costs for z_i under $\overline{y}$ for each $a_i \in \mathrm{dom}(z_i)$: $C_{z_i}(a_i, \overline{y})$

foreach $y_i \in Y$ **do** $C_{y_i}(\overline{y}[i], \overline{y}) \leftarrow 0$ **endfor**
foreach $z_i \in Z$ going from leaves to the root **do**
$C_{z_i}(a_i, \overline{y}) \leftarrow$
$\sum_{z_j \text{child of} z_i} \min_{a_j \in \mathrm{dom}(z_j)}(C_{z_j}(a_j, \overline{y}) + R_{z_i,z_j}(a_i, a_j))$
endfor
foreach $z_i \in Z$ going from the root to the leaves **do**
$\overline{z}[i] \leftarrow \arg\min_{a_i \in \mathrm{dom}(z_i)}(C_{z_i}(a_i, \overline{y}) + R_{z_i,z_{p_i}}(a_i, a_{p_i}))$
provided z_{p_i} is the parent of z_i
endfor
return $\overline{z}$

Properties of Stochastic Local Search

SLS algorithms ...

- are anytime: the longer the run, the better the solution they produce (in terms of a cost function counting violated constraints)
- terminate at local minima
- cannot be used to prove inconsistency of CSP instances

However, WalkSAT can be shown to find a satisfying assignment with probability approaching 1, provided the procedure can run long enough (exponentially long) and provided such an assignment exists.

Literature

Rina Dechter.
Constraint Processing,
Chapter 7, Morgan Kaufmann, 2003