Theoretical Computer Science II

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Exercise Sheet 14

Due: February 10, 2010

Exercise 14.1 (P, 1.5 + 0.5 marks)

(a) Show that P is closed under union, intersection, and complement.

(b) The complexity class coP contains all languages L whose complement is in P. Formally, $coP = \{L \mid \bar{L} \in P\}$. Is $P = coP$?

Exercise 14.2 (NP, 2 + 2 marks)

(a) The language $DOUBLESAT$ is defined as

$$DOUBLESAT = \{\langle\phi\rangle \mid \phi \text{ is a propositional formula that has at least two satisfying assignments}\}.$$

Show that $DOUBLESAT \in NP$.

(b) The language $SETPACK$ is defined as

$$SETPACK = \{\langle C, k\rangle \mid C \text{ is a finite collection of finite sets, with at least } k \text{ sets being disjoint}\}.$$

Show that $SETPACK \in NP$.

Exercise 14.3 (Reduction, 2 + 2 marks)

A *clique* in an undirected graph is a subset of its vertices such that every two vertices in the subset are connected by an edge. An *independent set* in an undirected graph is a subset of its vertices such that for any two vertices in the subset there is no edge connecting them. A *vertex cover* of an undirected graph is a set of vertices such that each edge of the graph is incident to at least one vertex of the set. Consider the following three languages:

$$\begin{aligned} CLIQUE &= \{\langle G, k\rangle \mid G \text{ is an undirected graph that contains a clique with } k \text{ vertices}\} \\ INDSET &= \{\langle G, k\rangle \mid G \text{ is an undirected graph that contains an independent set with } k \text{ vertices}\} \\ VERTEXCOVER &= \{\langle G, k\rangle \mid G \text{ is an undirected graph that has a vertex cover with } k \text{ vertices}\} \end{aligned}$$

(a) Show that $CLIQUE$ is polynomially reducible to $INDSET$.

(b) Show that $INDSET$ is polynomially reducible to $VERTEXCOVER$.