Theoretical Computer Science II

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Exercise Sheet 12

Due: January 27, 2010

Exercise 12.1 (Decidable Languages, 0.5 + 0.5 + 0.5 + 0.5 marks)

Consider the DFA $M = (Q, \Sigma, \delta, q_0, F)$ with $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, $F = \{q_0\}$ and δ being given by the table:

δ	0	1
q_0	q_0	q_1
q_1	q_2	q_2
q_2	q_0	q_1

(a) Is $\langle M, 0100\rangle \in A_{DFA}$?

(b) Is $\langle M, 011\rangle \in A_{DFA}$?

(c) Is $\langle M\rangle \in E_{DFA}$?

(d) Is $\langle M, M\rangle \in EQ_{DFA}$?

Justify your answers!

Exercise 12.2 (Decidable Languages, 1.5 + 1.5 + 1.5 marks)

Show that the following languages are decidable:

(a) $EQ_{DFA\_RE} = \{\langle D, R\rangle \mid D \text{ is a DFA and } R \text{ is a regular expression and } L(D) = L(R)\}$

(b) $A_{\epsilon CFG} = \{\langle G\rangle \mid G \text{ is a CFG that generates } \epsilon\}$

(c) $ALL_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA that recognizes } \Sigma^*\}$

Exercise 12.3 (Undecidable Languages, 4 marks)

Consider the problem of determining whether a two-tape Turing machine ever writes a non-blank symbol on its second tape, i.e.

$$N = \{\langle M, w\rangle \mid M \text{ is a two-tape Turing machine which writes a non-blank symbol onto its second tape when it runs on } w\}.$$

Show that N is undecidable. *Hint*: Use a reduction from A_{TM}.