Theoretical Computer Science II

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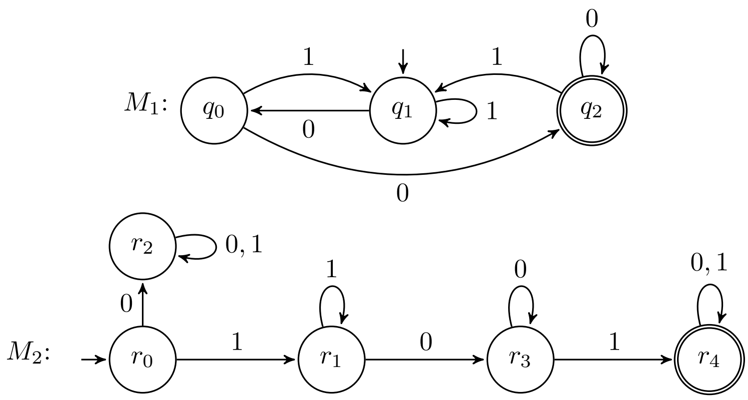
Exercise Sheet 6

Due: December 2, 2009

Exercise 6.1 (DFA, 1+1+2 marks)

Consider the following two DFAs (deterministic finite automata) with $\Sigma = \{0, 1\}$:



(a) What languages (L_1 and L_2) do these two automata individually recognize?

(b) Give the formal definition for M_1.

(c) Show that $L_1 \cup L_2$ is also a regular language, by constructing **one** DFA. Please hand in a **high quality** diagram.

Hint: Use the definition of the union automaton from the proof that the regular languages are closed under the union operation and remove all states (and transitions) that are not reachable from the start state.

Exercise 6.2 (DFA and NFA, 1+1 marks)

(a) Construct a DFA that recognizes the language $L = \{w \mid \text{ the length of } w \text{ is at most } 3\}$ with an alphabet $\Sigma = \{0, 1\}$:.

(b) Give the state diagram for an NFA (non-deterministic finite automaton) with at most four states accepting the language $L = \{w \mid w \text{ ends with 01 or with 10}\}$. The alphabet is $\Sigma = \{0, 1\}$.

Exercise 6.3 (Regular Languages, 2.5 + 1.5 marks)

In this exercise we want to prove that regular languages are closed under intersection and under complement. The intersection of two languages is defined as $L_1 \cap L_2$. The complement of a language is defined as the set of all words in Σ^* which are not in L, i.e. $\bar{L} = \Sigma^* \setminus L$ (Σ^* is the set of all words/strings over Σ).

Let L and L' be regular languages that are recognized by DFAs $M = (Q, \Sigma, \delta, q_0, F)$ and $M' = (Q', \Sigma', \delta', q_0', F')$, respectively.

(a) Show that the regular languages are closed under *intersection*, i.e. give a finite automaton that recognizes $L \cap L'$.

(b) Show that the regular languages are closed under *complement*, i.e. give a finite automaton that recognizes $\bar{L}$.