**Theoretical Computer Science II**

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# Exercise Sheet 5

**Due: November 25, 2009**

**Exercise 5.1** (Predicate Logic – Free and bound variables, 2 marks)

Mark the free occurrences of variables in the following formulae and specify the set of free variables for each of these formulae. (In the underlying signature, $x, y$ and $z$ are variable symbols and a and b are constant symbols.)

(a) $\mathrm{P}(\mathrm{a}, y)$

(b) $\mathrm{P}(x, y) \wedge \exists x \forall z(\mathrm{Q}(x, y, z))$

(c) $\exists x(\mathrm{P}(x, y) \wedge \mathrm{Q}(x)) \vee \mathrm{P}(y, x)$

(d) $\forall x(\exists y(\mathrm{P}(x, y) \wedge \mathrm{Q}(x)) \vee \mathrm{P}(x, y))$

(e) $\forall x \forall y(\mathrm{P}(x, \mathrm{b}) \wedge \mathrm{Q}(x) \vee \mathrm{P}(\mathrm{f}(y), x))$

(f) $\exists x(\exists z(\mathrm{R}(x, z)) \vee \forall y(\mathrm{S}(y, \mathrm{g}(x, z))))$

(g) $\exists x(\exists z(\mathrm{R}(x, z))) \vee \forall y(\mathrm{S}(y, \mathrm{g}(x, z)))$

**Exercise 5.2** (Entailment in Predicate Logic, 3 marks)

Prove that $\forall x \exists y \mathrm{P}(x, y) \models \exists x \mathrm{P}(\mathrm{f}(z), x)$.

**Exercise 5.3** (Equivalences in Predicate Logic, 2.5 + 2.5 marks)

(a) Prove the equivalence $\neg \forall x \phi \equiv \exists x \neg \phi$.

(b) Show that in general it does *not* hold that $\forall x \exists y \phi \equiv \exists y \forall x \phi$ by giving a counter example.