## Theoretical Computer Science II

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# Exercise Sheet 4

**Due: November 18, 2009**

**Exercise 4.1** (Predicate Logic – Terminology, 2 marks)

Classify the following expressions as *terms*, *ground terms*, *atoms*, *formulae*, or *meta language* (statements that are not part of predicate logic itself but statements about the semantics). If there is more than one possibility for an expression, please list them all. In the expressions, a and b are constant symbols, $x$ and $y$ are variable symbols, f and g are function symbols, and P and Q are relation symbols.

(a) $\mathrm{P}(x, y)$

(b) $\mathrm{f}(\mathrm{a}, \mathrm{b})$

(c) $\mathcal{I} \models \mathrm{P}(\mathrm{a}, \mathrm{f}(\mathrm{b}))$

(d) $\mathcal{I}, \alpha \models \mathrm{P}(\mathrm{a}, \mathrm{f}(x))$

(e) $\mathrm{f}(\mathrm{g}(x), \mathrm{b})$

(f) $\mathrm{Q}(x)$ is satisfiable.

(g) $\exists x(\mathrm{P}(x, y) \wedge \mathrm{Q}(x)) \vee \mathrm{P}(y, x)$

(h) $\forall x(\exists y(\mathrm{P}(x, y) \wedge \mathrm{Q}(x)) \vee \mathrm{P}(x, y))$

(i) $\forall x \forall y(\mathrm{P}(x, y) \wedge \mathrm{Q}(x) \vee \mathrm{P}(\mathrm{f}(y), x))$

(j) $\mathrm{Q}(x) \vee \mathrm{P}(x, y) \equiv \mathrm{P}(x, y) \vee \mathrm{Q}(x)$

**Exercise 4.2** (Formalization in Predicate Logic, 3 marks)

Let L be a binary and P und O be two unary relation symbols. Further, let t be a constant symbol and s be a unary function symbol. The intended semantics of the symbols is given by the interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{D} = \mathbb{N}$, $\mathrm{L}^{\mathcal{I}} = <$, $\mathrm{P}^{\mathcal{I}} = \{n \in \mathbb{N} \mid n \text{ is prime}\}$, $\mathrm{O}^{\mathcal{I}} = \{n \in \mathbb{N} \mid n \text{ is odd}\}$, $\mathrm{t}^{\mathcal{I}} = 3$ and $\mathrm{s}^{\mathcal{I}}(n) = n + 1$ for all $n \in \mathbb{N}$. Symbolize the following statements:

(a) Not all natural numbers are prime.

(b) There is a prime number different from three.

(c) For each prime number $n$, the number $n + 1$ is not prime, unless $n + 1 = 3$.

(d) There is exactly one even prime number.

(e) There are infinitely many prime numbers.

(f) There is a smallest natural number.

**Exercise 4.3** (Predicate Logic – Interpretations, 4+1 marks)

Consider the following set of formulae:

$$KB = \left\{ \begin{array}{l} \forall x \neg \mathrm{P}(x, x) \\ \forall x \forall y \forall z((\mathrm{P}(x, y) \wedge \mathrm{P}(y, z)) \rightarrow \mathrm{P}(x, z)) \\ \forall x \forall y(\mathrm{P}(x, y) \vee x = y \vee \mathrm{P}(y, x)) \end{array} \right\}$$

(a) Specify an interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{D} = \{d_1, \ldots, d_4\}$ and prove that $\mathcal{I} \models KB$ (i.e., $\mathcal{I} \models \varphi$ for all $\varphi \in KB$). Why is it not necessary to specify a variable assignment $\alpha$ to state a model of $KB$?

(b) Are there also models of $KB$ with an infinite domain $\mathcal{D}$? If yes, give such an interpretation. If not, justify your answer.