

# Principles of AI Planning

## 12. State-space search: merge-and-shrink abstractions

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# Principles of AI Planning

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## Motivation

Limitations of pattern databases

Main ideas

Running example

## Synchronized products

Definition

Example

Properties

## Algorithm

Merge steps and shrink steps

Abstraction mapping

Concrete algorithm

## Conclusion

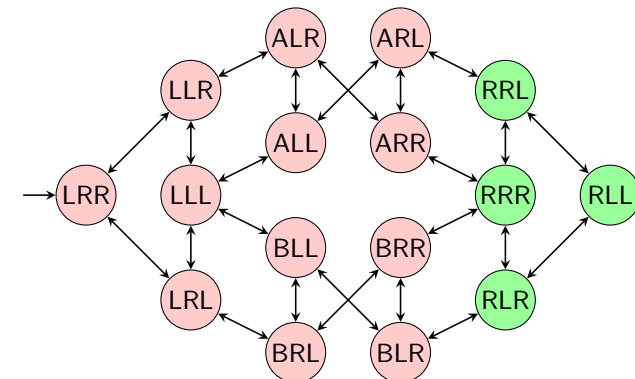
Theoretical properties and practical performance

Literature

## Beyond pattern databases

- ▶ Despite their popularity, pattern databases have some **fundamental limitations** (↔ example on next slides).
- ▶ In this chapter, we study a recently introduced class of abstractions called **merge-and-shrink abstractions**.
- ▶ Merge-and-shrink abstractions can be seen as a **proper generalization** of pattern databases.
  - ▶ They can do everything that pattern databases can do (modulo polynomial extra effort).
  - ▶ They can do some things that pattern databases cannot.
- ▶ Initial experiments with merge-and-shrink abstractions have shown very promising results.
- ▶ They have provably greater **representational power** than pattern databases for many common planning domains.

## Back to the running example

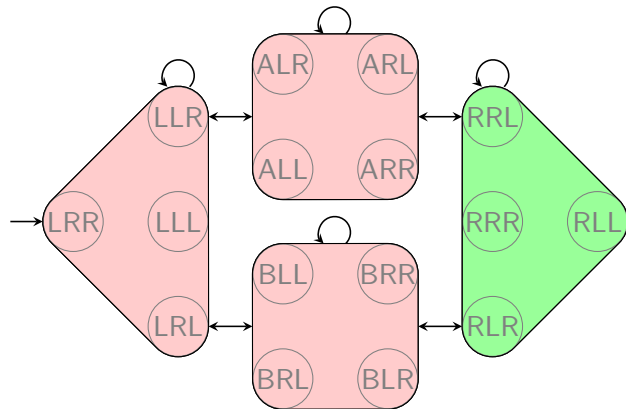


Logistics problem with one package, two trucks, two locations:

- ▶ state variable **package**:  $\{L, R, A, B\}$
- ▶ state variable **truck A**:  $\{L, R\}$
- ▶ state variable **truck B**:  $\{L, R\}$

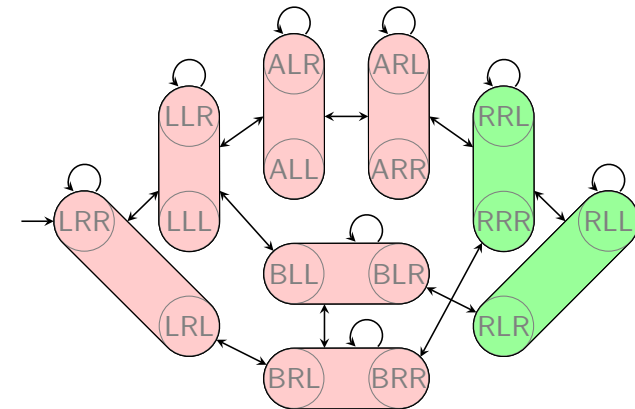
## Example: projection

Project to {package}:



## Example: projection (2)

Project to {package, truck A}:



## Limitations of projections

How accurate is the PDB heuristic?

- ▶ consider **generalization of the example**:  
 $N$  trucks,  $M$  locations (fully connected), still one package
- ▶ consider **any** pattern that is proper subset of variable set  $V$
- ▶  $h(s_0) \leq 2 \rightsquigarrow$  **no better** than atomic projection to **package**

These values cannot be improved by maximizing over several patterns or using additive patterns.

**Merge-and-shrink abstractions** can represent heuristics with  $h(s_0) \geq 3$  for tasks of this kind of any size.

Time and space requirements are **polynomial in  $N$  and  $M$** .

## Merge-and-shrink abstractions: main idea

### Main idea of merge-and-shrink abstractions

(due to Dräger, Finkbeiner & Podelski, 2006):

Instead of **perfectly** reflecting **a few** state variables, reflect **all** state variables, but in a **potentially lossy** way.

## The need for succinct abstraction mappings

- ▶ One major difficulty for non-PDB abstractions is to **succinctly represent the abstraction mapping**.
- ▶ For pattern databases, this is easy because the abstraction mappings – projections – are very **structured**.
- ▶ For less rigidly structured abstraction mappings, we need another idea.

## Merge-and-shrink abstractions: idea

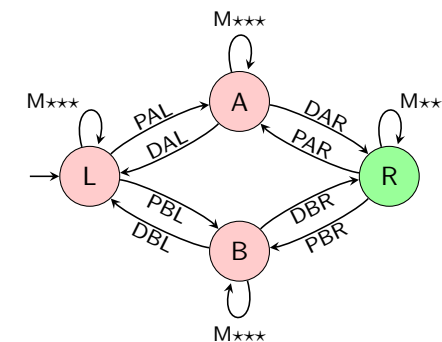
- ▶ The main idea underlying merge-and-shrink abstractions is that given two abstractions  $\mathcal{A}$  and  $\mathcal{A}'$ , we can **merge** them into a new **product abstraction**.
- ▶ The product abstraction **captures all information** of both abstractions and can be **better informed than either**.
- ▶ It can even be better informed than their **sum**.
- ▶ By merging a set of very simple abstractions, we can in theory represent **arbitrary** abstractions of an SAS<sup>+</sup> task.
- ▶ In practice, due to memory limitations, such abstractions can become too large. In that case, we can **shrink** them by abstracting them further using **any abstraction** on an intermediate result, then **continue the merging process**.

## Running example: explanations

- ▶ **Atomic projections** – projections to a single state variable – play an important role in this chapter.
- ▶ Unlike previous chapters, **transition labels** are critically important in this chapter.
- ▶ Hence we now look at the transition systems for atomic projections of our example task, including transition labels.
- ▶ We abbreviate operator names as in these examples:
  - ▶ **MALR**: move truck **A** from **left** to **right**
  - ▶ **DAR**: drop package from truck **A** at **right** location
  - ▶ **PBL**: pick up package with truck **B** at **left** location
- ▶ We abbreviate parallel arcs with **commas** and **wildcards (\*)** in the labels as in these examples:
  - ▶ **PAL, DAL**: two parallel arcs labeled **PAL** and **DAL**
  - ▶ **MA\*\***: two parallel arcs labeled **MALR** and **MARL**

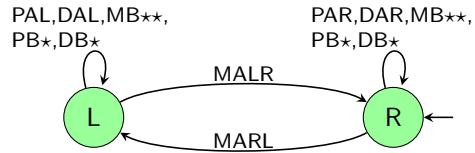
## Running example: atomic projection for package

$\mathcal{T}^{\pi_{\{\text{package}\}}}$ :



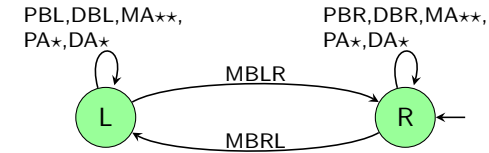
## Running example: atomic projection for truck A

$\mathcal{T}^\pi\{\text{truck A}\}$ :



## Running example: atomic projection for truck B

$\mathcal{T}^\pi\{\text{truck B}\}$ :



## Synchronized product of transition systems

### Definition (synchronized product of transition systems)

For  $i \in \{1, 2\}$ , let  $\mathcal{T}_i = \langle S_i, L, T_i, l_i, G_i \rangle$  be transition systems with identical label set.

The **synchronized product** of  $\mathcal{T}_1$  and  $\mathcal{T}_2$ , in symbols  $\mathcal{T}_1 \otimes \mathcal{T}_2$ , is the transition system  $\mathcal{T}_\otimes = \langle S_\otimes, L, T_\otimes, l_\otimes, G_\otimes \rangle$  with

- ▶  $S_\otimes := S_1 \times S_2$
- ▶  $T_\otimes := \{ \langle \langle s_1, s_2 \rangle, l, \langle t_1, t_2 \rangle \rangle \mid \langle s_1, l, t_1 \rangle \in T_1 \text{ and } \langle s_2, l, t_2 \rangle \in T_2 \}$
- ▶  $l_\otimes := l_1 \times l_2$
- ▶  $G_\otimes := G_1 \times G_2$

## Synchronized product of functions

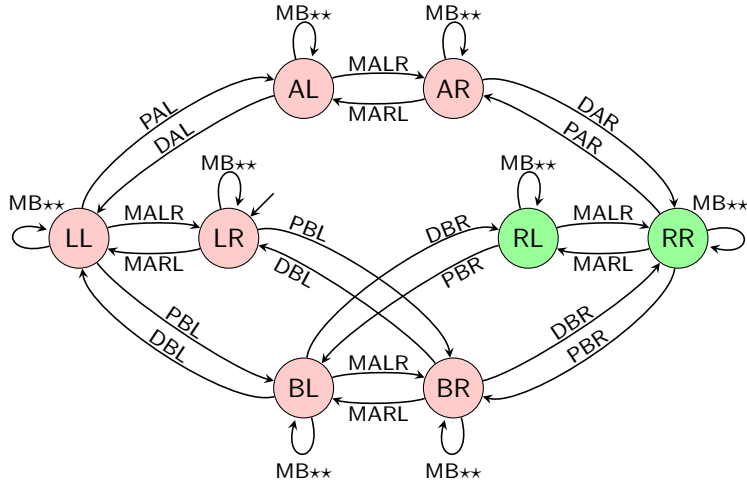
### Definition (synchronized product of functions)

Let  $\alpha_1 : S \rightarrow S_1$  and  $\alpha_2 : S \rightarrow S_2$  be functions with identical domain.

The **synchronized product** of  $\alpha_1$  and  $\alpha_2$ , in symbols  $\alpha_1 \otimes \alpha_2$ , is the function  $\alpha_\otimes : S \rightarrow S_1 \times S_2$  defined as  $\alpha_\otimes(s) = \langle \alpha_1(s), \alpha_2(s) \rangle$ .

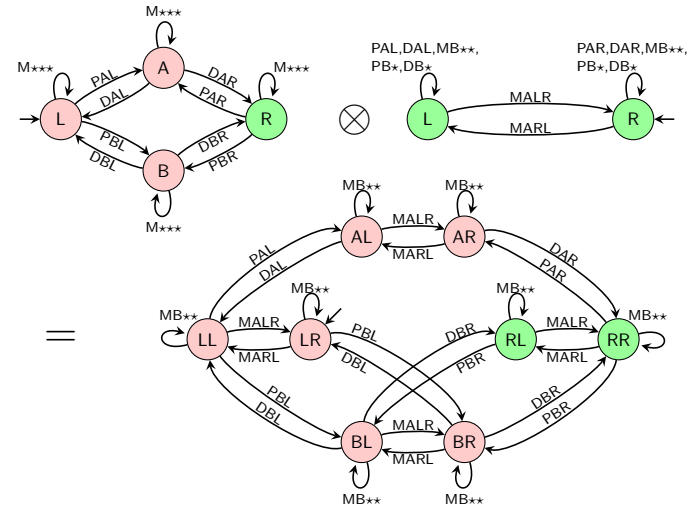
### Example: synchronized product

$$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$$



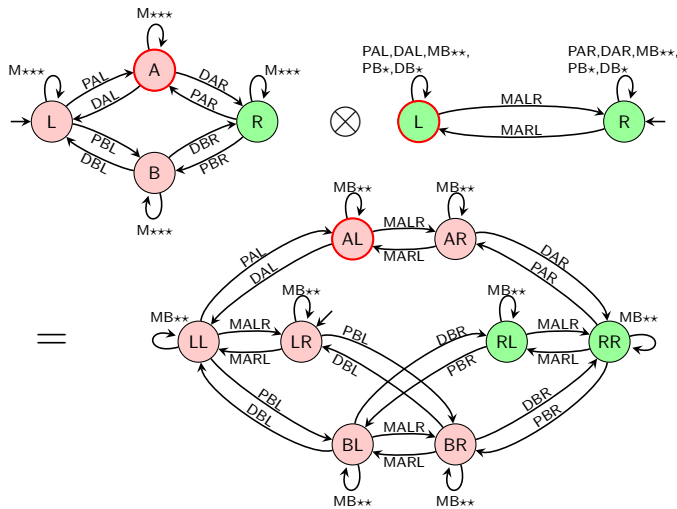
### Example: computation of synchronized product

$$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}:$$



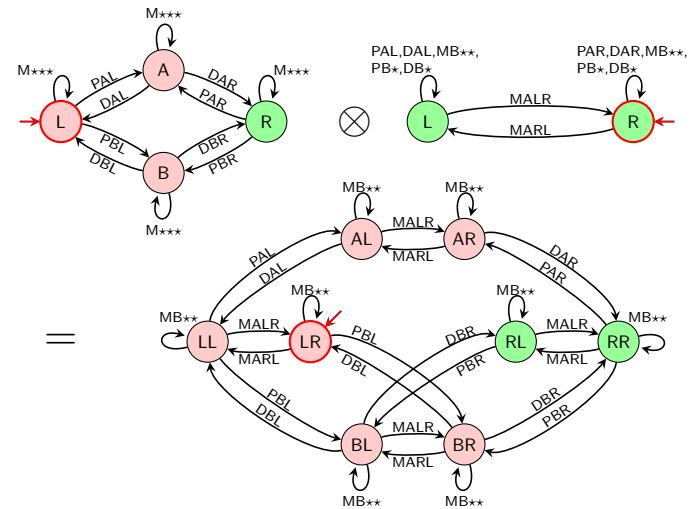
### Example: computation of synchronized product

$$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}: S_\otimes = S_1 \times S_2$$



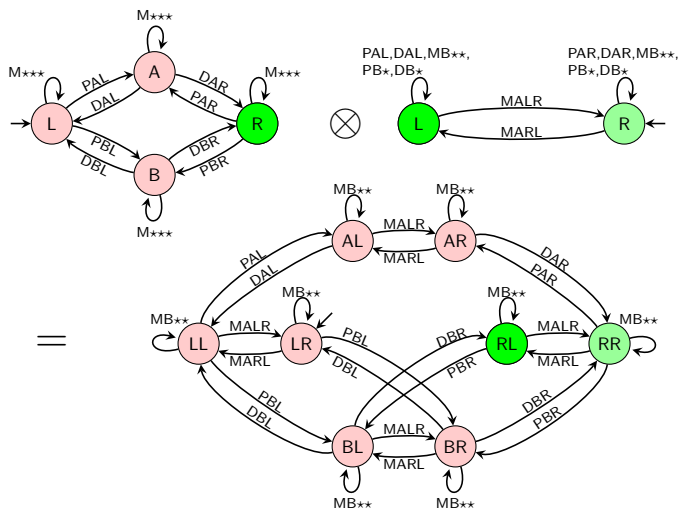
### Example: computation of synchronized product

$$\mathcal{T}^\pi\{\text{package}\} \otimes \mathcal{T}^\pi\{\text{truck A}\}: I_\otimes = I_1 \times I_2$$



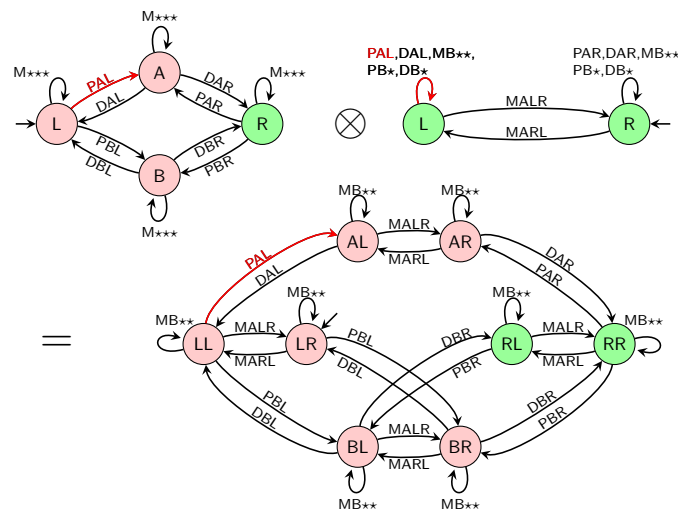
### Example: computation of synchronized product

$$\mathcal{T}^\pi_{\{\text{package}\}} \otimes \mathcal{T}^\pi_{\{\text{truck A}\}} : G_\otimes = G_1 \times G_2$$



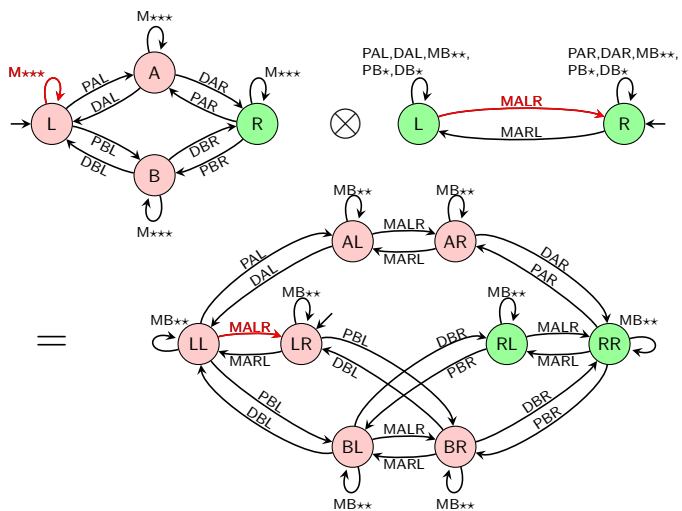
### Example: computation of synchronized product

$$\mathcal{T}^\pi_{\{\text{package}\}} \otimes \mathcal{T}^\pi_{\{\text{truck A}\}} : T_\otimes := \{ \langle \langle s_1, s_2 \rangle, l, \langle t_1, t_2 \rangle \rangle \mid \dots \}$$



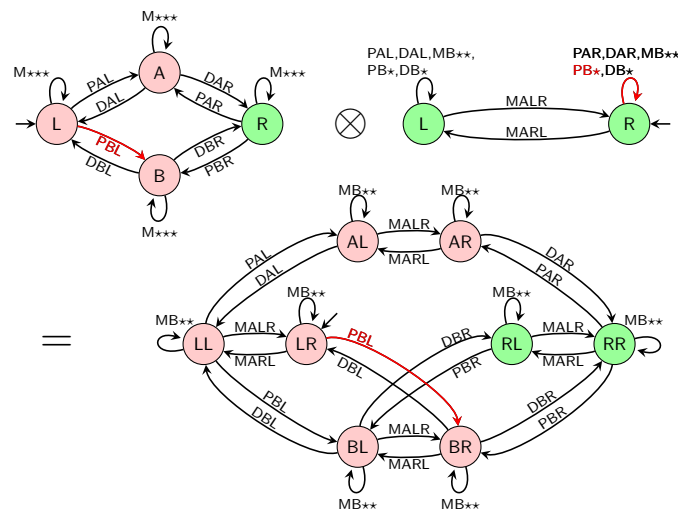
### Example: computation of synchronized product

$$\mathcal{T}^\pi_{\{\text{package}\}} \otimes \mathcal{T}^\pi_{\{\text{truck A}\}} : T_\otimes := \{ \langle \langle s_1, s_2 \rangle, l, \langle t_1, t_2 \rangle \rangle \mid \dots \}$$



### Example: computation of synchronized product

$$\mathcal{T}^\pi_{\{\text{package}\}} \otimes \mathcal{T}^\pi_{\{\text{truck A}\}} : T_\otimes := \{ \langle \langle s_1, s_2 \rangle, l, \langle t_1, t_2 \rangle \rangle \mid \dots \}$$









Recovering  $\mathcal{T}(\Pi)$  from the atomic projections

- ▶ By repeated application of the corollary, we can recover **all pattern database abstractions** of an SAS<sup>+</sup> planning task from the abstractions for atomic projections.
- ▶ In particular, by computing the product of **all** atomic projections, we can recover the abstraction for the **identity abstraction**  $\text{id} = \pi_V$ .

Corollary (Recovering  $\mathcal{T}(\Pi)$  from the atomic projections)

Let  $\Pi$  be an SAS<sup>+</sup> planning task with variable set  $V$ .

Then  $\mathcal{T}(\Pi) \sim \bigotimes_{v \in V} \mathcal{T}^{\pi_{\{v\}}}$ .

- ▶ This is an important result because it shows that the abstractions for atomic projections **contain all information** of an SAS<sup>+</sup> task.

## Generic merge-and-shrink abstractions: outline

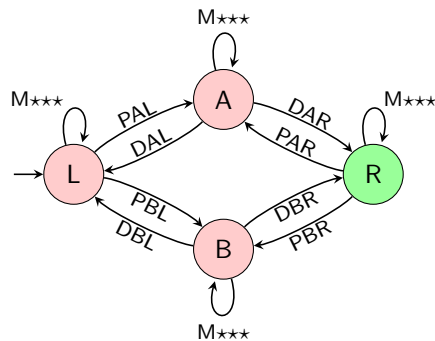
Using the results from the previous section, we can develop the ideas of a **generic abstraction computation procedure** that **takes all state variables into account**:

- ▶ **Initialization step**: Compute all abstract transition systems for atomic projections to form the initial abstraction collection.
- ▶ **Merge steps**: Combine two abstractions in the collection by replacing them with their synchronized product. (Stop once only one abstraction is left.)
- ▶ **Shrink steps**: If the abstractions in the collection are too large to compute their synchronized product, make them smaller by abstracting them further (applying an arbitrary homomorphism to them).

We explain these steps with our running example.

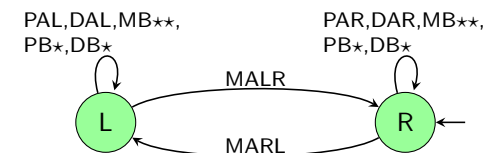
## Initialization step: atomic projection for package

$\mathcal{T}^{\pi_{\{\text{package}\}}}$ :

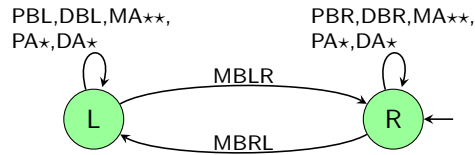


## Initialization step: atomic projection for truck A

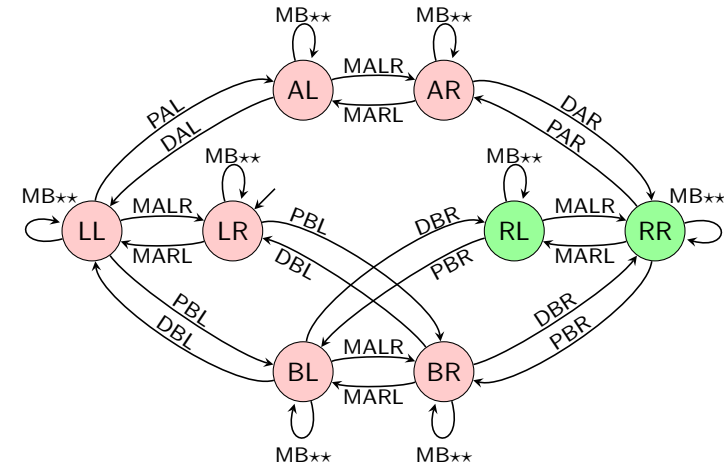
$\mathcal{T}^{\pi_{\{\text{truck A}\}}}$ :



## Initialization step: atomic projection for truck B

 $\mathcal{T}^{\pi}\{\text{truck B}\}$ :current collection:  $\{\mathcal{T}^{\pi}\{\text{package}\}, \mathcal{T}^{\pi}\{\text{truck A}\}, \mathcal{T}^{\pi}\{\text{truck B}\}\}$ 

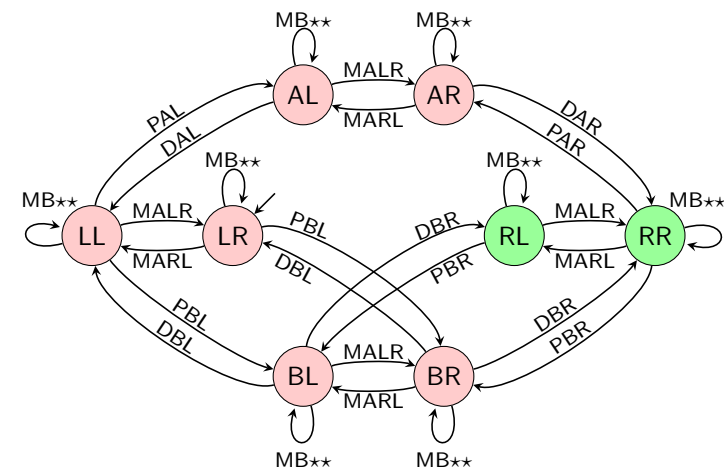
## First merge step

 $\mathcal{T}_1 := \mathcal{T}^{\pi}\{\text{package}\} \otimes \mathcal{T}^{\pi}\{\text{truck A}\}$ :current collection:  $\{\mathcal{T}_1, \mathcal{T}^{\pi}\{\text{truck B}\}\}$ 

## Need to simplify?

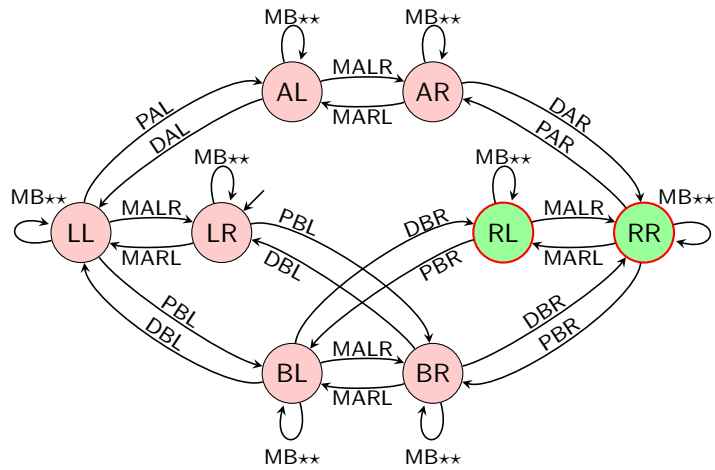
- ▶ If we have sufficient memory available, we can now compute  $\mathcal{T}_1 \otimes \mathcal{T}^{\pi}\{\text{truck B}\}$ , which would recover the complete transition system of the task.
- ▶ However, to illustrate the general idea, let us assume that we do not have sufficient memory for this product.
- ▶ More specifically, we will assume that after each product operation we need to reduce the result abstraction to **four states** to obey memory constraints.
- ▶ So we need to reduce  $\mathcal{T}_1$  to four states. We have a lot of leeway in deciding **how exactly** to abstract  $\mathcal{T}_1$ .
- ▶ In this example, we simply use an abstraction that leads to a good result in the end.

## First shrink step

 $\mathcal{T}_2 := \text{some abstraction of } \mathcal{T}_1$ 

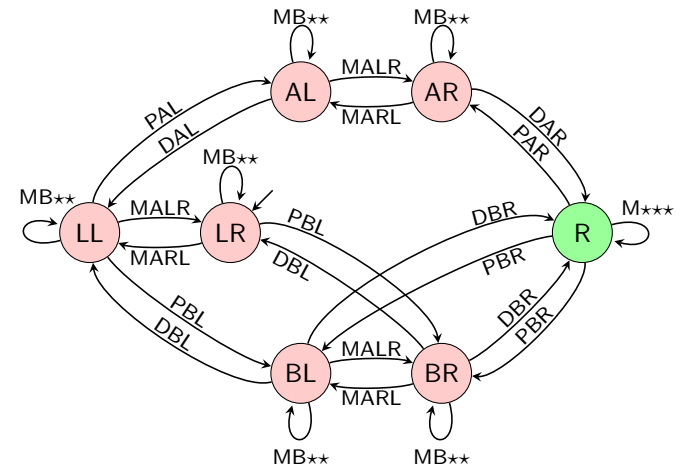
### First shrink step

$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



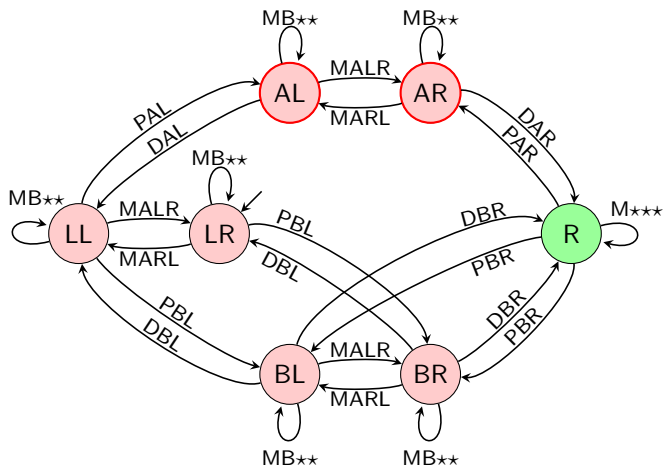
### First shrink step

$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



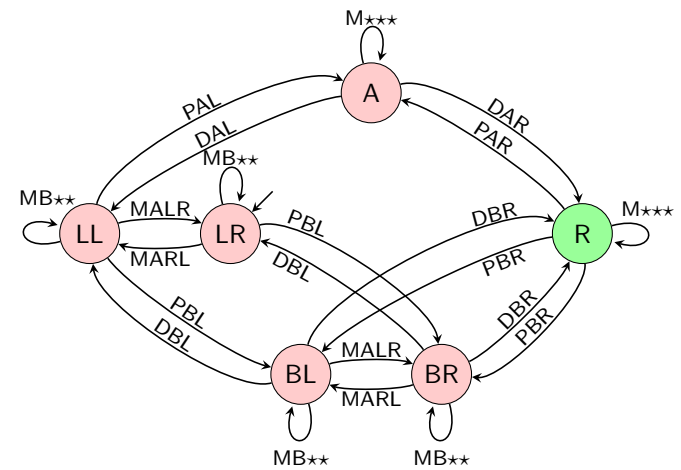
### First shrink step

$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



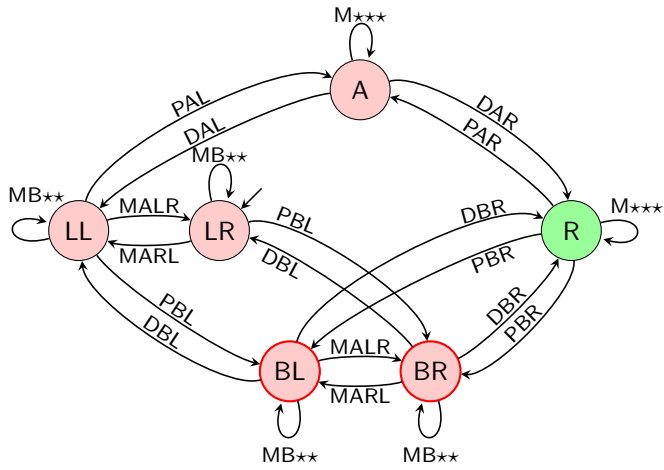
### First shrink step

$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



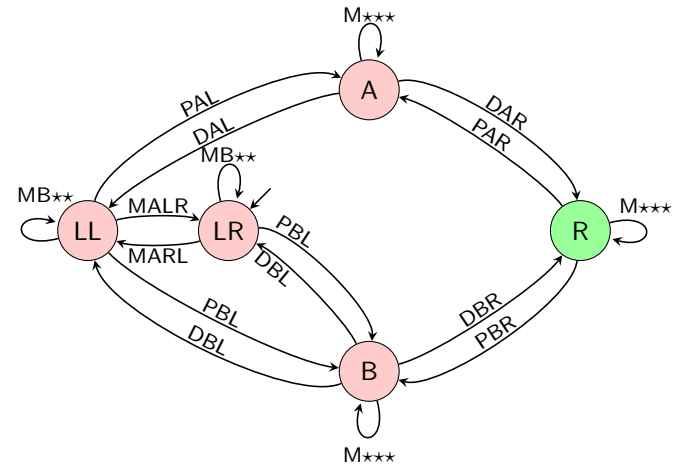
### First shrink step

$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



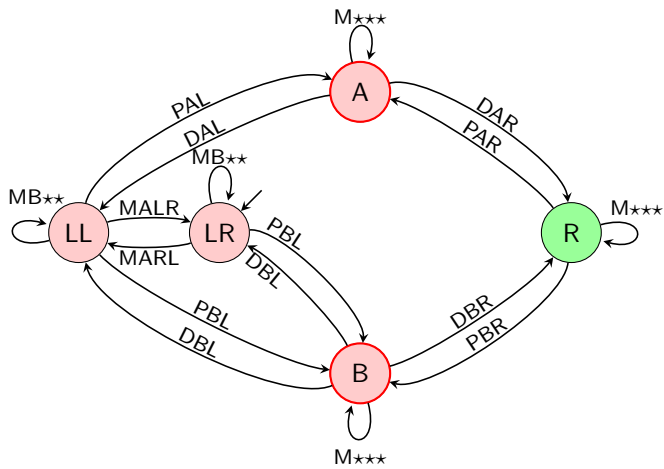
### First shrink step

$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



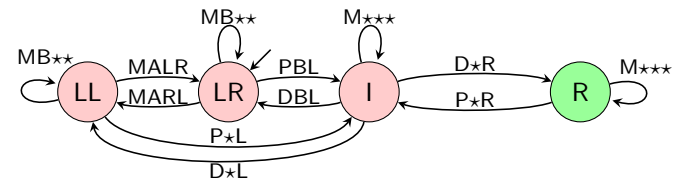
### First shrink step

$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



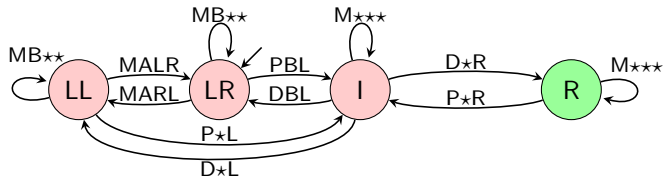
### First shrink step

$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



### First shrink step

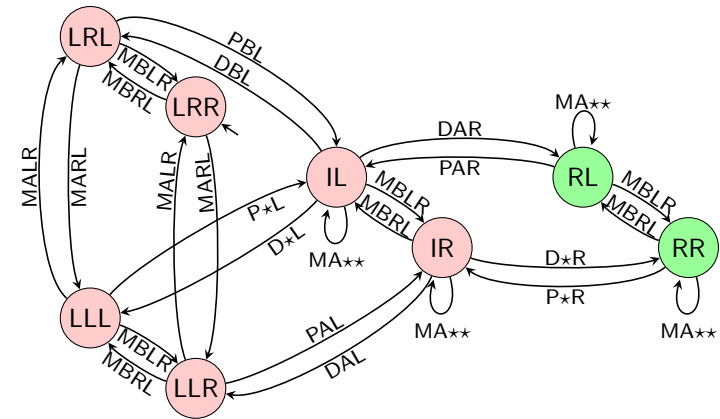
$\mathcal{T}_2 :=$  some abstraction of  $\mathcal{T}_1$



current collection:  $\{\mathcal{T}_2, \mathcal{T}^{\pi\{\text{truck B}\}}\}$

### Second merge step

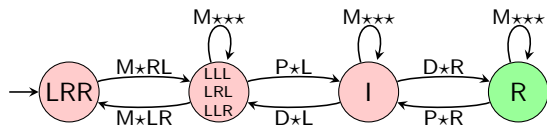
$\mathcal{T}_3 := \mathcal{T}_2 \otimes \mathcal{T}^{\pi\{\text{truck B}\}}$ :



current collection:  $\{\mathcal{T}_3\}$

### Another shrink step?

- ▶ Normally we could stop now and use the distances in the final abstraction as our heuristic function.
- ▶ However, if there were further state variables to integrate, we would simplify further, e.g. leading to the following abstraction (again with four states):



- ▶ We get a heuristic value of 3 for the initial state, **better than any PDB heuristic** that is a proper abstraction.
- ▶ The example generalizes to more locations and trucks, even if we stick to the size limit of 4 (after merging).

### How to represent the abstraction mapping?

**Idea:** the computation of the abstraction mapping follows the sequence of product computations

- ▶ For the **atomic abstractions** for  $\pi_{\{v\}}$ , we generate a **one-dimensional table** that denotes which value in  $\mathcal{D}_v$  corresponds to which abstract state.
- ▶ During the **merge** (product) step  $\mathcal{A} := \mathcal{A}_1 \otimes \mathcal{A}_2$ , we generate a **two-dimensional table** that denotes which pair of states of  $\mathcal{A}_1$  and  $\mathcal{A}_2$  corresponds to which state of  $\mathcal{A}$ .
- ▶ During the **shrink** (abstraction) steps, we make sure to keep the table in sync with the abstraction choices.

## How to represent the abstraction mapping? (ctd.)

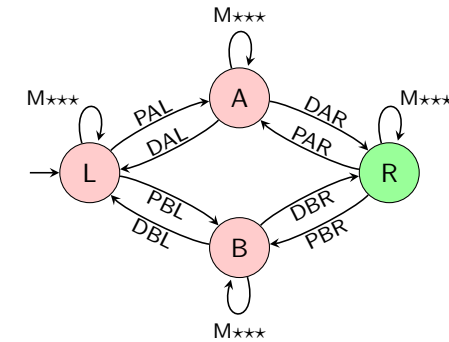
**Idea:** the computation of the abstraction mapping follows the sequence of product computations

- ▶ Once we have computed the final abstraction, we compute all **abstract goal distances** and store them in a **one-dimensional table**.
- ▶ At this point, we can **throw away** all the abstractions – we just need to keep the tables.
- ▶ During **search**, we do a sequence of table lookups to navigate from the atomic abstraction states to the final abstraction state and heuristic value  
 $\rightsquigarrow 2|V|$  lookups,  $O(|V|)$  time

Again, we illustrate the process with our running example.

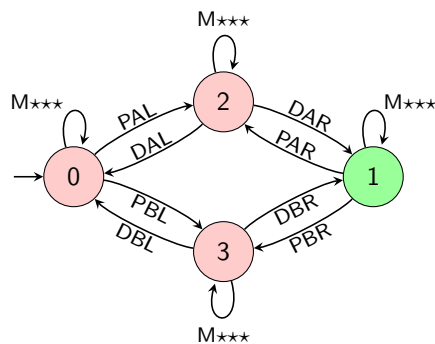
## Abstraction mapping example: atomic abstractions

Computing abstraction mappings for the atomic abstractions is simple. Just number the states (domain values) consecutively and generate a table of references to the states:



## Abstraction mapping example: atomic abstractions

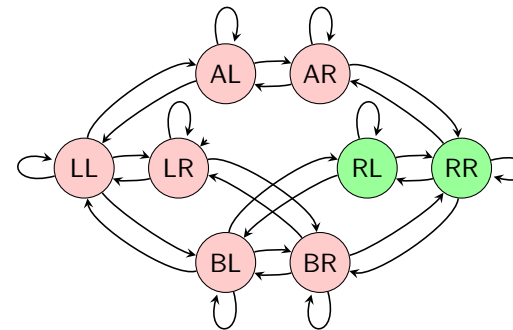
Computing abstraction mappings for the atomic abstractions is simple. Just number the states (domain values) consecutively and generate a table of references to the states:



L	R	A	B
0	1	2	3

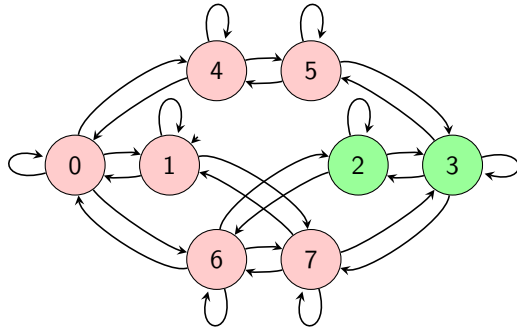
## Abstraction mapping example: merge step

For product abstractions  $\mathcal{A}_1 \otimes \mathcal{A}_2$ , we again number the product states consecutively and generate a table that links state pairs of  $\mathcal{A}_1$  and  $\mathcal{A}_2$  to states of  $\mathcal{A}$ :



## Abstraction mapping example: merge step

For product abstractions  $\mathcal{A}_1 \otimes \mathcal{A}_2$ , we again number the product states consecutively and generate a table that links state pairs of  $\mathcal{A}_1$  and  $\mathcal{A}_2$  to states of  $\mathcal{A}$ :



	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	3
$s_1 = 2$	4	5
$s_1 = 3$	6	7

## Maintaining the mapping when shrinking

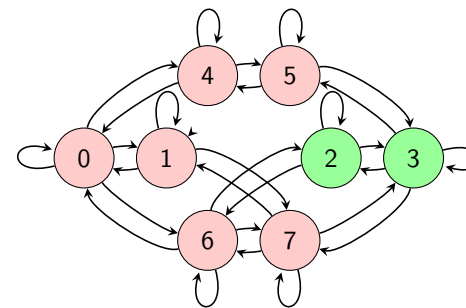
- ▶ The hard part in representing the abstraction mapping is to keep it consistent when shrinking.
- ▶ In theory, this is easy to do:
  - ▶ When combining states  $i$  and  $j$ , arbitrarily use one of them (say  $i$ ) as the number of the new state.
  - ▶ Find all table entries in the table for this abstraction which map to the other state  $j$  and change them to  $i$ .
- ▶ However, doing a table scan each time two states are combined is very inefficient.
- ▶ Fortunately, there also is an efficient implementation which takes constant time per combination.

## Maintaining the mapping efficiently

- ▶ Associate each abstract state with a linked list, representing **all table entries that map to this state**.
- ▶ Before starting the shrink operation, initialize the lists by scanning through the table, then **discard the table**.
- ▶ While shrinking, when combining  $i$  and  $j$ , **splice the list elements of  $j$  into the list elements of  $i$** .
  - ▶ For linked lists, this is a **constant-time operation**.
- ▶ Once shrinking is completed, renumber all abstract states so that there are no gaps in the numbering.
- ▶ Finally, regenerate the mapping table from the linked list information.

## Abstraction mapping example: shrink step

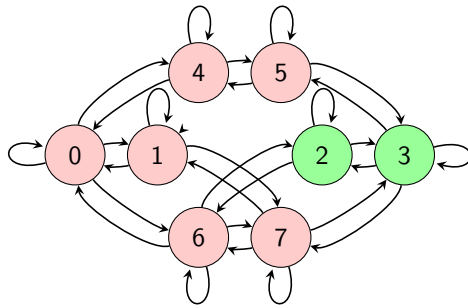
Representation before shrinking:



	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	3
$s_1 = 2$	4	5
$s_1 = 3$	6	7

## Abstraction mapping example: shrink step

1. Convert table to linked lists and discard it.

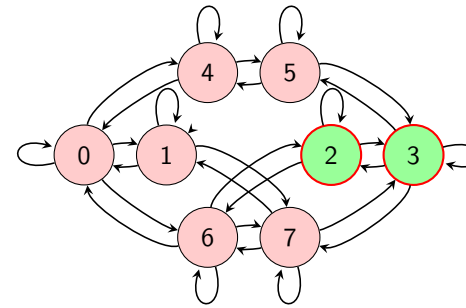


$list_0 = \{(0, 0)\}$   
 $list_1 = \{(0, 1)\}$   
 $list_2 = \{(1, 0)\}$   
 $list_3 = \{(1, 1)\}$   
 $list_4 = \{(2, 0)\}$   
 $list_5 = \{(2, 1)\}$   
 $list_6 = \{(3, 0)\}$   
 $list_7 = \{(3, 1)\}$

	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	3
$s_1 = 2$	4	5
$s_1 = 3$	6	7

## Abstraction mapping example: shrink step

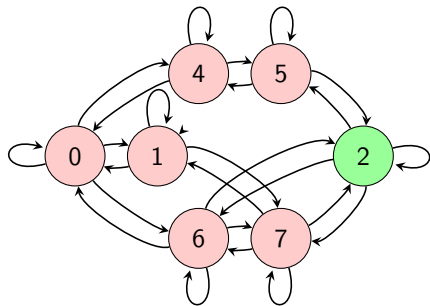
2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .



$list_0 = \{(0, 0)\}$   
 $list_1 = \{(0, 1)\}$   
 $list_2 = \{(1, 0)\}$   
 $list_3 = \{(1, 1)\}$   
 $list_4 = \{(2, 0)\}$   
 $list_5 = \{(2, 1)\}$   
 $list_6 = \{(3, 0)\}$   
 $list_7 = \{(3, 1)\}$

## Abstraction mapping example: shrink step

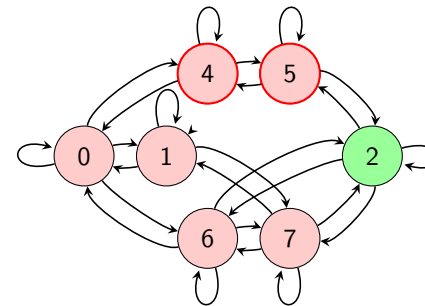
2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .



$list_0 = \{(0, 0)\}$   
 $list_1 = \{(0, 1)\}$   
 $list_2 = \{(1, 0), (1, 1)\}$   
 $list_3 = \emptyset$   
 $list_4 = \{(2, 0)\}$   
 $list_5 = \{(2, 1)\}$   
 $list_6 = \{(3, 0)\}$   
 $list_7 = \{(3, 1)\}$

## Abstraction mapping example: shrink step

2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .

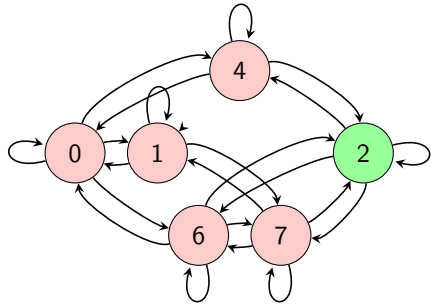


$list_0 = \{(0, 0)\}$   
 $list_1 = \{(0, 1)\}$   
 $list_2 = \{(1, 0), (1, 1)\}$   
 $list_3 = \emptyset$   
 $list_4 = \{(2, 0)\}$   
 $list_5 = \{(2, 1)\}$   
 $list_6 = \{(3, 0)\}$   
 $list_7 = \{(3, 1)\}$



## Abstraction mapping example: shrink step

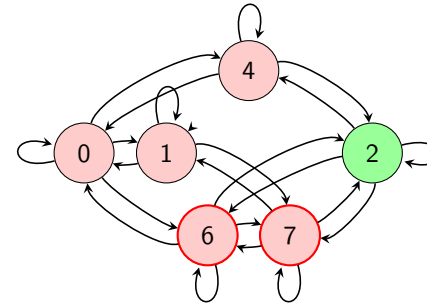
2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .



$list_0 = \{(0, 0)\}$   
 $list_1 = \{(0, 1)\}$   
 $list_2 = \{(1, 0), (1, 1)\}$   
 $list_3 = \emptyset$   
 $list_4 = \{(2, 0), (2, 1)\}$   
 $list_5 = \emptyset$   
 $list_6 = \{(3, 0)\}$   
 $list_7 = \{(3, 1)\}$

## Abstraction mapping example: shrink step

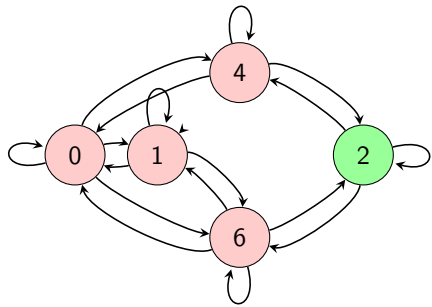
2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .



$list_0 = \{(0, 0)\}$   
 $list_1 = \{(0, 1)\}$   
 $list_2 = \{(1, 0), (1, 1)\}$   
 $list_3 = \emptyset$   
 $list_4 = \{(2, 0), (2, 1)\}$   
 $list_5 = \emptyset$   
 $list_6 = \{(3, 0)\}$   
 $list_7 = \{(3, 1)\}$

## Abstraction mapping example: shrink step

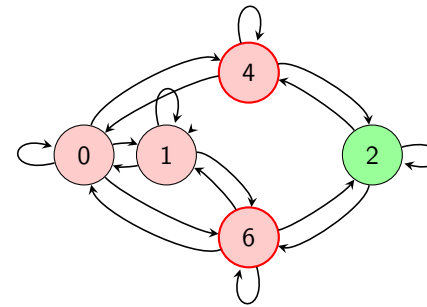
2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .



$list_0 = \{(0, 0)\}$   
 $list_1 = \{(0, 1)\}$   
 $list_2 = \{(1, 0), (1, 1)\}$   
 $list_3 = \emptyset$   
 $list_4 = \{(2, 0), (2, 1)\}$   
 $list_5 = \emptyset$   
 $list_6 = \{(3, 0), (3, 1)\}$   
 $list_7 = \emptyset$

## Abstraction mapping example: shrink step

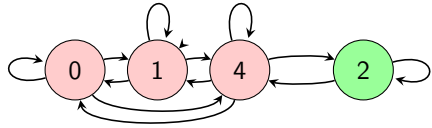
2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .



$list_0 = \{(0, 0)\}$   
 $list_1 = \{(0, 1)\}$   
 $list_2 = \{(1, 0), (1, 1)\}$   
 $list_3 = \emptyset$   
 $list_4 = \{(2, 0), (2, 1)\}$   
 $list_5 = \emptyset$   
 $list_6 = \{(3, 0), (3, 1)\}$   
 $list_7 = \emptyset$

## Abstraction mapping example: shrink step

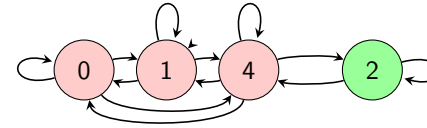
2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .



$$\begin{aligned} list_0 &= \{(0, 0)\} \\ list_1 &= \{(0, 1)\} \\ list_2 &= \{(1, 0), (1, 1)\} \\ list_3 &= \emptyset \\ list_4 &= \{(2, 0), (2, 1), \\ &\quad (3, 0), (3, 1)\} \\ list_5 &= \emptyset \\ list_6 &= \emptyset \\ list_7 &= \emptyset \end{aligned}$$

## Abstraction mapping example: shrink step

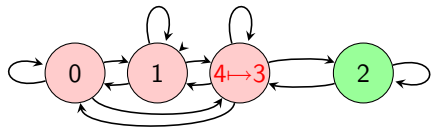
2. When combining  $i$  and  $j$ , splice  $list_j$  into  $list_i$ .



$$\begin{aligned} list_0 &= \{(0, 0)\} \\ list_1 &= \{(0, 1)\} \\ list_2 &= \{(1, 0), (1, 1)\} \\ list_3 &= \emptyset \\ list_4 &= \{(2, 0), (2, 1), \\ &\quad (3, 0), (3, 1)\} \\ list_5 &= \emptyset \\ list_6 &= \emptyset \\ list_7 &= \emptyset \end{aligned}$$

## Abstraction mapping example: shrink step

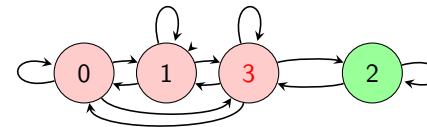
3. Renumber abstract states consecutively.



$$\begin{aligned} list_0 &= \{(0, 0)\} \\ list_1 &= \{(0, 1)\} \\ list_2 &= \{(1, 0), (1, 1)\} \\ list_3 &= \emptyset \\ list_4 &= \{(2, 0), (2, 1), \\ &\quad (3, 0), (3, 1)\} \\ list_5 &= \emptyset \\ list_6 &= \emptyset \\ list_7 &= \emptyset \end{aligned}$$

## Abstraction mapping example: shrink step

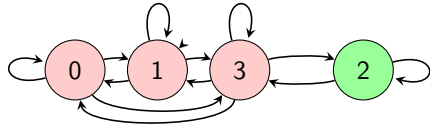
3. Renumber abstract states consecutively.



$$\begin{aligned} list_0 &= \{(0, 0)\} \\ list_1 &= \{(0, 1)\} \\ list_2 &= \{(1, 0), (1, 1)\} \\ list_3 &= \{(2, 0), (2, 1), \\ &\quad (3, 0), (3, 1)\} \\ list_4 &= \emptyset \\ list_5 &= \emptyset \\ list_6 &= \emptyset \\ list_7 &= \emptyset \end{aligned}$$

## Abstraction mapping example: shrink step

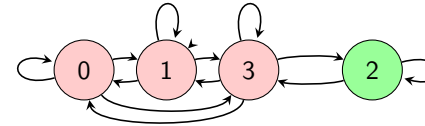
## 4. Regenerate the mapping table from the linked lists.



$$\begin{aligned} list_0 &= \{(0, 0)\} \\ list_1 &= \{(0, 1)\} \\ list_2 &= \{(1, 0), (1, 1)\} \\ list_3 &= \{(2, 0), (2, 1), \\ &\quad (3, 0), (3, 1)\} \\ list_4 &= \emptyset \\ list_5 &= \emptyset \\ list_6 &= \emptyset \\ list_7 &= \emptyset \end{aligned}$$

## Abstraction mapping example: shrink step

## 4. Regenerate the mapping table from the linked lists.



$$\begin{aligned} list_0 &= \{(0, 0)\} \\ list_1 &= \{(0, 1)\} \\ list_2 &= \{(1, 0), (1, 1)\} \\ list_3 &= \{(2, 0), (2, 1), \\ &\quad (3, 0), (3, 1)\} \\ list_4 &= \emptyset \\ list_5 &= \emptyset \\ list_6 &= \emptyset \\ list_7 &= \emptyset \end{aligned}$$

	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1
$s_1 = 1$	2	2
$s_1 = 2$	3	3
$s_1 = 3$	3	3

## The final heuristic representation

At the end, our heuristic is represented by six tables:

- ▶ three one-dimensional tables for the atomic abstractions:

$T_{\text{package}}$	L	R	A	B	$T_{\text{truck A}}$	L	R	$T_{\text{truck B}}$	L	R
	0	1	2	3		0	1		0	1

- ▶ two tables for the two merge and subsequent shrink steps:

$T_{m\&s}^1$	$s_2 = 0$	$s_2 = 1$	$T_{m\&s}^2$	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	0	1	$s_1 = 0$	1	1
$s_1 = 1$	2	2	$s_1 = 1$	1	0
$s_1 = 2$	3	3	$s_1 = 2$	2	2
$s_1 = 3$	3	3	$s_1 = 3$	3	3

- ▶ one table with goal distances for the final abstraction:

$T_h$	$s = 0$	$s = 1$	$s = 2$	$s = 3$
$h(s)$	3	2	1	0

Given a state  $s = \{\text{package} \mapsto p, \text{truck A} \mapsto a, \text{truck B} \mapsto b\}$ , its heuristic value is then looked up as:

- ▶  $h(s) = T_h[T_{m\&s}^2[T_{m\&s}^1[T_{\text{package}}[p], T_{\text{truck A}}[a]], T_{\text{truck B}}[b]]]$

## Towards a concrete algorithm

- ▶ We have now described how merge-and-shrink abstractions work in general.
- ▶ However, we have not said how exactly to decide
  - ▶ which abstractions to combine in a merge step and
  - ▶ when and how to further abstract in a shrink step.
- ▶ There are many possibilities here (just like there are many possible PDB heuristics).
- ▶ Only one concrete method, called  $h_{HHH}$ , has been explored so far in planning, which we will now discuss briefly.

## Generic algorithm template

### Generic abstraction computation algorithm

$abs := \{\mathcal{T}^{\pi\{v\}} \mid v \in V\}$

**while**  $abs$  contains more than one abstraction:

**select**  $\mathcal{A}_1, \mathcal{A}_2$  from  $abs$

**shrink**  $\mathcal{A}_1$  and/or  $\mathcal{A}_2$  until  $size(\mathcal{A}_1) \cdot size(\mathcal{A}_2) \leq N$

$abs := abs \setminus \{\mathcal{A}_1, \mathcal{A}_2\} \cup \{\mathcal{A}_1 \otimes \mathcal{A}_2\}$

**return** the remaining abstraction in  $abs$

$N$ : parameter bounding number of abstract states

### Questions for practical implementation:

- ▶ Which abstractions to select?  $\rightsquigarrow$  **merging strategy**
- ▶ How to shrink an abstraction?  $\rightsquigarrow$  **shrinking strategy**
- ▶ How to choose  $N$ ?  $\rightsquigarrow$  usually: as high as memory allows

## Merging strategy

### Which abstractions to select?

$h_{HHH}$ : Linear merging strategy

In each iteration after the first, choose the abstraction computed in the previous iteration as  $\mathcal{A}_1$ .

$\rightsquigarrow$  fully defined by an ordering of atomic projections

**Rationale:** only maintains one “complex” abstraction at a time

$h_{HHH}$ : Ordering of atomic projections

- ▶ Start with a goal variable.
- ▶ Add variables that appear in preconditions of operators affecting previous variables.
- ▶ If that is not possible, add a goal variable.

**Rationale:** increases  $h$  quickly (cf. causal graph criteria for growing patterns)

## Shrinking strategy

### Which abstractions to shrink?

$h_{HHH}$ : only shrink the product

If at all possible, don't shrink atomic abstractions, but only product abstractions.

**Rationale:** Product abstractions are more likely to contain significant redundancies and symmetries.

## Shrinking strategy (ctd.)

### How to shrink an abstraction?

$h_{HHH}$ :  $f$ -preserving shrinking strategy

Repeatedly combine abstract states with

**identical** abstract goal distances ( $h$  values) and  
**identical** abstract initial state distances ( $g$  values).

**Rationale:** preserves heuristic value and overall graph shape

$h_{HHH}$ : Tie-breaking criterion

Prefer combining states where  $g + h$  is high.

In case of ties, combine states where  $h$  is high.

**Rationale:** states with high  $g + h$  values are less likely to be explored by  $A^*$ , so inaccuracies there matter less

## Properties of merge-and-shrink abstractions

- ▶ We conclude by briefly mentioning a number of **theoretical** properties of merge-and-shrink abstractions (without proof).
- ▶ While these theoretical results are interesting, heuristics in planning usually need to be justified by good **empirical performance**.
- ▶ Regarding empirical performance, initial results for  $h_{HHH}$  are **very encouraging**, outperforming pattern databases (and all other admissible heuristics) on a number of benchmark domains.
- ▶ However, merge-and-shrink abstractions are **not nearly as well studied** (and understood) as pattern databases, so the **jury is still out**.

## Theoretical properties: as good as PDBs

### As powerful as PDBs

**Pattern database heuristics** are a **special case** of our abstraction heuristics, and arise naturally as a side product.

- ▶ More precisely, PDB heuristics are merge-and-shrink abstractions without shrink steps (terminating heuristic computation as soon as space runs out).
- ▶ However, specialized PDB algorithms are faster than the generic merge-and-shrink algorithm.
- ▶ This performance difference is only **polynomial**, but this does not mean that it does not matter in practice!
- ▶ Still, this shows that **representational power** is **at least as large** as that of PDB heuristics.

## Theoretical properties: better than PDBs

### Greater representational power

In **some planning domains** where polynomial-sized pattern database heuristics have unbounded error (Gripper, Schedule, two Promela variants), merge-and-shrink abstractions can obtain **perfect heuristics** in polynomial time with suitable merging/shrinking strategies.

- ▶ This shows that **representational power** is **strictly greater** than that of PDB heuristics.
- ▶ However, it does **not** mean that we know good general (domain-independent) merging/shrinking strategies that will generate these perfect heuristics in practice.

## Theoretical properties: additivity

### Get additivity for free

If  $P_1$  and  $P_2$  are **additive patterns** of a  $SAS^+$  task, then for **all**  $h$ -preserving merge-and-shrink abstractions  $\mathcal{A}_1$  of  $\mathcal{T}^{\pi P_1}$ ,  $\mathcal{A}_2$  of  $\mathcal{T}^{\pi P_2}$  and  $\mathcal{A}$  of  $\mathcal{A}_1 \otimes \mathcal{A}_2$ , the abstraction heuristic for  $\mathcal{A}$  **dominates**  $h^{P_1} + h^{P_2}$ . (An abstraction is  **$h$ -preserving** if  $\alpha(s) = \alpha(s')$  only for  $s, s'$  with same abstract goal distance.)

- ▶ One can derive a similar **theory of additivity** for merge-and-shrink abstraction as for pattern databases.
- ▶ However, this result shows that this is **not as necessary** as for pattern databases: additivity is **exploited automatically** by a single merge-and-shrink abstraction to some extent.
- ▶ Still, experimental results show that there is sometimes a benefit of using multiple merge-and-shrink abstractions. (However, so far only maximization has been explored.)

## Literature

References on merge-and-shrink abstractions:



Klaus Dräger, Bernd Finkbeiner and Andreas Podelski.  
Directed Model Checking with Distance-Preserving Abstractions.  
*Proc. SPIN 2006*, pp. 19–34, 2006.  
Introduces merge-and-shrink abstractions (for model-checking).



Malte Helmert, Patrik Haslum and Jörg Hoffmann.  
Flexible Abstraction Heuristics for Optimal Sequential Planning.  
*Proc. ICAPS 2007*, pp. 176–183, 2007.  
Introduces merge-and-shrink abstractions for planning.  
Most ideas of this chapter come from this paper.