Principles of AI Planning

2. Transition systems

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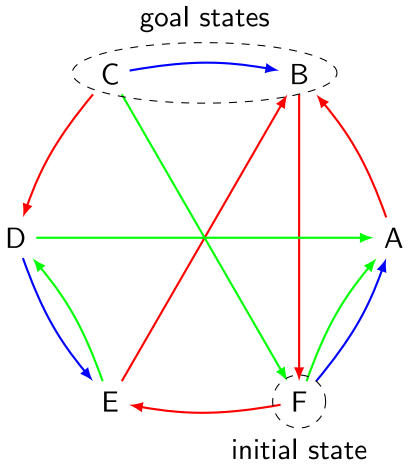
Transition systems



Transition systems

Formalization of the dynamics of the world/application

Definition (transition system)

A transition system is $\langle S, I, \{a_1, \ldots, a_n\}, G \rangle$ where

- S is a finite set of states (the state space),
- $I \subseteq S$ is a finite set of initial states,
- every action $a_i \subseteq S \times S$ is a binary relation on S,
- $G \subseteq S$ is a finite set of goal states.

Definition (applicable action)

An action a is applicable in a state s if sas' for at least one state s'.

A transition system is deterministic if there is only one initial state and all actions are deterministic. Hence all future states of the world are completely predictable.

Definition (deterministic transition system)

A deterministic transition system is $\langle S, I, O, G \rangle$ where

- S is a finite set of states (the state space),
- $I \in S$ is a state,
- actions $a \in O$ (with $a \subseteq S \times S$) are partial functions,
- $G \subseteq S$ is a finite set of goal states.

Successor state wrt. an action

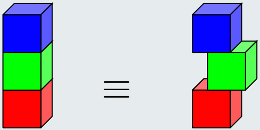
Given a state s and an action a so that a is applicable in s, the successor state of s with respect to a is s' such that sas', denoted by $s' = app_a(s)$.

Blocks world

The rules of the game

Location on the table does not matter.

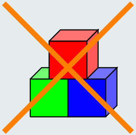
Location on a block does not matter.



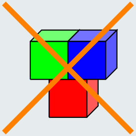
Blocks world

The rules of the game

At most one block may be below a block.

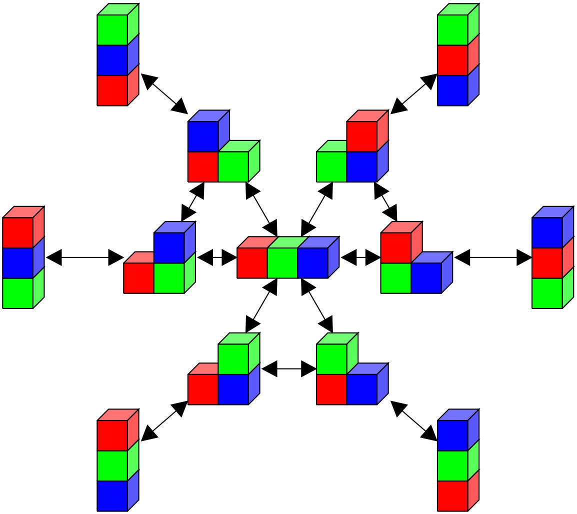


At most one block may be on top of a block.



Blocks world

The transition graph for three blocks



Blocks world
Properties

blocks	states
1	1
2	3
3	13
4	73
5	501
6	4051
7	37633
8	394353
9	4596553
10	58941091

1. Finding a solution is polynomial time in the number of blocks (move everything onto the table and then construct the goal configuration).
2. Finding a shortest solution is NP-complete (for a compact description of the problem).

Deterministic planning: plans

Definition (plan)

A plan for $\langle S, I, O, G \rangle$ is a sequence $\pi = o_1, \dots, o_n$ of operators such that $o_1, \dots, o_n \in O$ and $s_0, \dots, s_n$ is a sequence of states (the execution of π) so that

1. $s_0 = I$,
2. $s_i = app_{o_i}(s_{i-1})$ for every $i \in \{1, \dots, n\}$, and
3. $s_n \in G$.

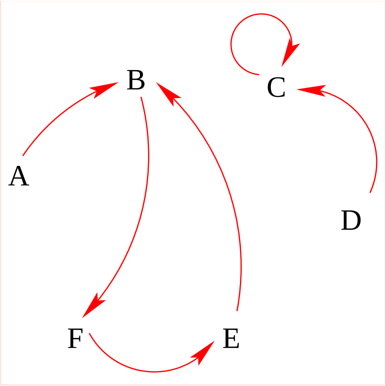
This can be equivalently expressed as

$$app_{o_n}(app_{o_{n-1}}(\dots app_{o_1}(I) \dots)) \in G$$

Transition relations as matrices

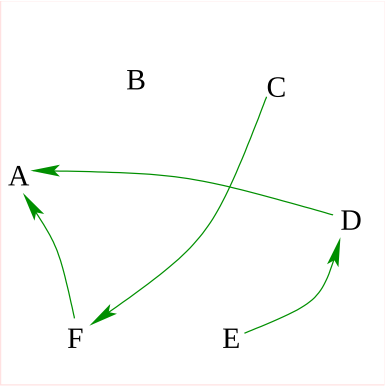
1. If there are n states, each action (a binary relation) corresponds to an $n \times n$ matrix: The element at row i and column j is 1 if the action maps state i to state j, and 0 otherwise.
   For deterministic actions there is at most one non-zero element in each row.
2. Matrix multiplication corresponds to sequential composition: taking action M_1 followed by action M_2 is the product $M_1 M_2$. (This also corresponds to the join of the relations.)
3. The unit matrix $I_{n \times n}$ is the NO-OP action: every state is mapped to itself.

Example



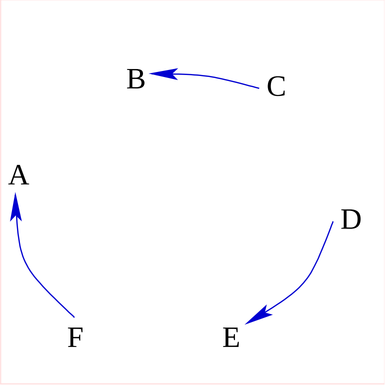
	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	0	0	0	1
C	0	0	1	0	0	0
D	0	0	1	0	0	0
E	0	1	0	0	0	0
F	0	0	0	0	1	0

Example



	A	B	C	D	E	F
A	0	0	0	0	0	0
B	0	0	0	0	0	0
C	0	0	0	0	0	1
D	1	0	0	0	0	0
E	0	0	0	1	0	0
F	1	0	0	0	0	0

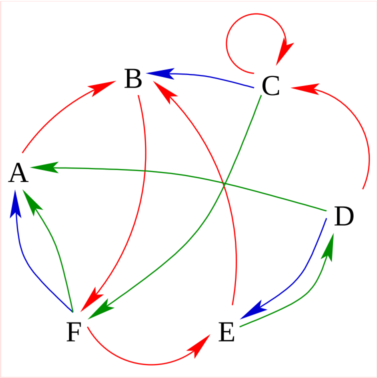
Example



	A	B	C	D	E	F
A	0	0	0	0	0	0
B	0	0	0	0	0	0
C	0	1	0	0	0	0
D	0	0	0	0	1	0
E	0	0	0	0	0	0
F	1	0	0	0	0	0

Sum matrix $M_R + M_G + M_B$

Representing one-step reachability by any of the component actions



	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	0	0	0	1
C	0	1	1	0	0	1
D	1	0	1	0	1	0
E	0	1	0	1	0	0
F	1	0	0	0	1	0

We use addition $0 + 0 = 0$ and $b + b' = 1$ if $b = 1$ or $b' = 1$.

Sequential composition as matrix multiplication

$$
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\ \hline
0 & 0 & 0 & 0 & 0 & \mathbf{1} \\ \hline
0 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
\times
\left(\begin{array}{cccc|c|c}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & \mathbf{1} & 0
\end{array}\right)
=
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & \mathbf{1} & 0 \\
1 & 1 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0
\end{pmatrix}
$$

E is reachable from B by two actions
because
F is reachable from B by one action and
E is reachable from F by one action.

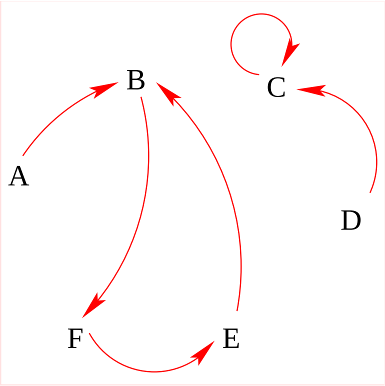
Reachability

Let M be the $n \times n$ matrix that is the (Boolean) sum of the matrices of the individual actions. Define

$$\begin{array}{rcl} R_0 & = & I_{n \times n} \\ R_1 & = & I_{n \times n} + M \\ R_2 & = & I_{n \times n} + M + M^2 \\ R_3 & = & I_{n \times n} + M + M^2 + M^3 \\ & \vdots & \end{array}$$

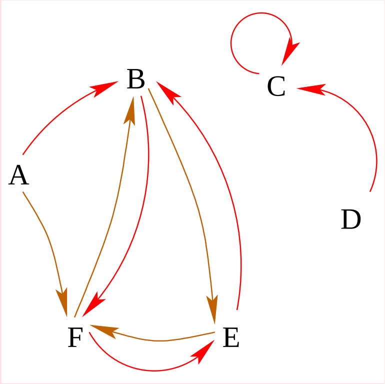
R_i represents reachability by i actions or less. If s' is reachable from s, then it is reachable with $\leq n - 1$ actions: $R_{n-1} = R_n$.

Reachability: example, M_R



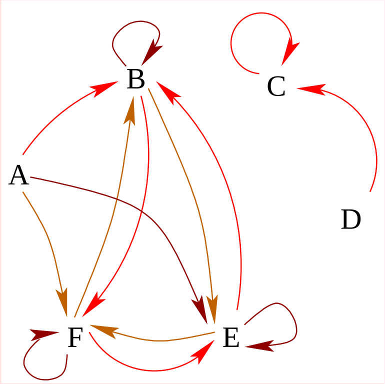
	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	0	0	0	1
C	0	0	1	0	0	0
D	0	0	1	0	0	0
E	0	1	0	0	0	0
F	0	0	0	0	1	0

Reachability: example, $M_R + M_R^2$



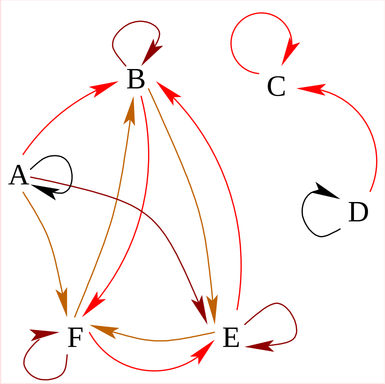
	A	B	C	D	E	F
A	0	1	0	0	0	1
B	0	0	0	0	1	1
C	0	0	1	0	0	0
D	0	0	1	0	0	0
E	0	1	0	0	0	1
F	0	1	0	0	1	0

Reachability: example, $M_R + M_R^2 + M_R^3$



	A	B	C	D	E	F
A	0	1	0	0	1	1
B	0	1	0	0	1	1
C	0	0	1	0	0	0
D	0	0	1	0	0	0
E	0	1	0	0	1	1
F	0	1	0	0	1	1

Reachability: example, $M_R + M_R^2 + M_R^3 + I_{6\times 6}$



	A	B	C	D	E	F
A	1	1	0	0	1	1
B	0	1	0	0	1	1
C	0	0	1	0	0	0
D	0	0	1	1	0	0
E	0	1	0	0	1	1
F	0	1	0	0	1	1

Relations and sets as matrices
Row vectors as sets of states

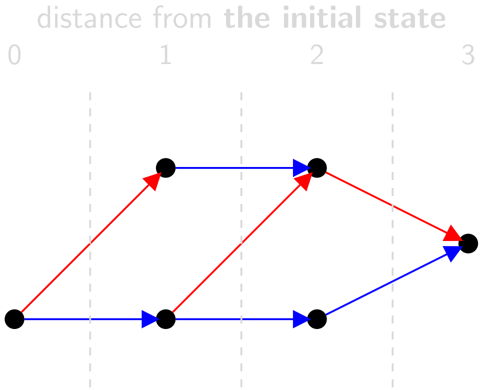
Row vectors S represent sets of states.
SM is the set of states reachable from S by M.

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T \times \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}^T$$

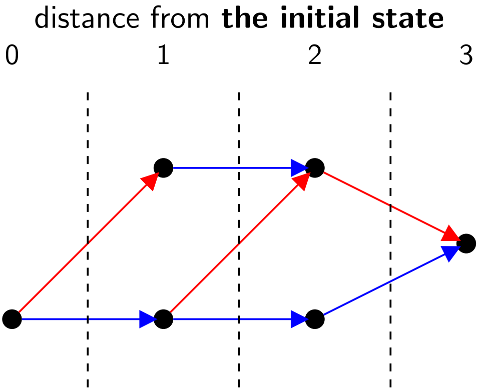
A simple planning algorithm

- We next present a simple planning algorithm based on computing distances in the transition graph.
- The algorithm finds shortest paths less efficiently than Dijkstra's algorithm; we present the algorithm because we later will use it as a basis of an algorithm that is applicable to much bigger state spaces than Dijkstra's algorithm directly.

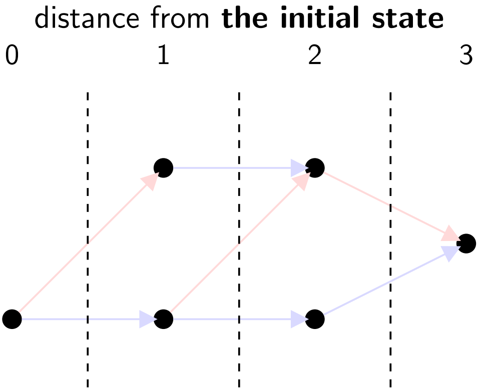
A simple planning algorithm
Idea



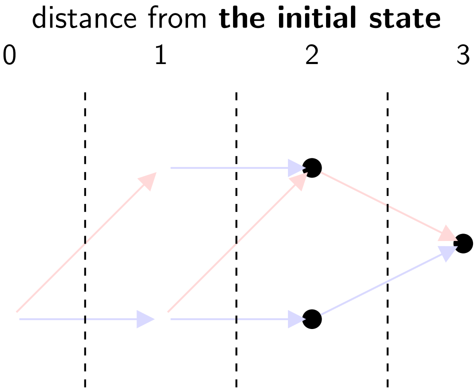
A simple planning algorithm
Idea



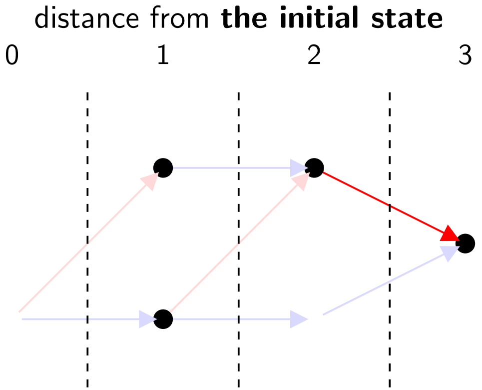
A simple planning algorithm
Idea



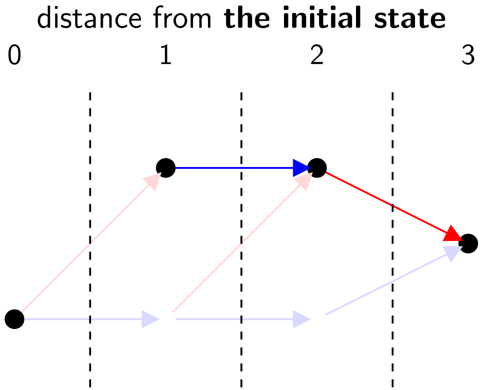
A simple planning algorithm
Idea



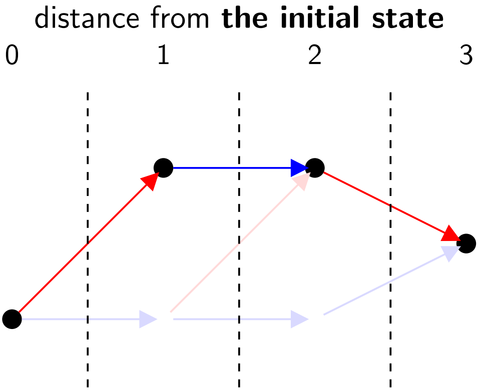
A simple planning algorithm
Idea



A simple planning algorithm
Idea



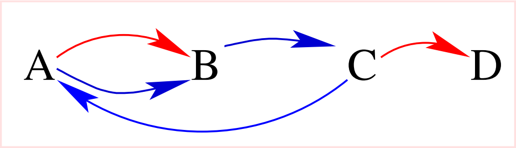
A simple planning algorithm
Idea



A simple planning algorithm

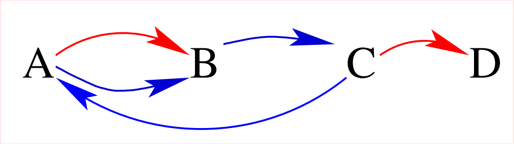
1. Compute the matrices $R_0, R_1, R_2, \ldots, R_n$ representing reachability with $0, 1, 2, \ldots, n$ steps with all actions.
2. Find the smallest i such that a goal state s_g is reachable from the initial state according to R_i.
3. Find an action (the last action of the plan) by which s_g is reached with one step from a state $s_{g'}$ that is reachable from the initial state according to R_{i-1}.
4. Repeat the last step, now viewing $s_{g'}$ as the goal state with distance $i - 1$.

Example



$$
\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 1 \\ D & 0 & 0 & 0 & 0 \end{array}
+
\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 & 0 \\ D & 0 & 0 & 0 & 0 \end{array}
=
\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 & 1 \\ D & 0 & 0 & 0 & 0 \end{array}
$$

Example



R_0 =

	A	B	C	D
A	1	0	0	0
B	0	1	0	0
C	0	0	1	0
D	0	0	0	1

R_1 =

	A	B	C	D
A	1	1	0	0
B	0	1	1	0
C	1	0	1	1
D	0	0	0	1

R_2 =

	A	B	C	D
A	1	1	1	0
B	1	1	1	1
C	1	1	1	1
D	0	0	0	1

R_3 =

	A	B	C	D
A	1	1	1	1
B	1	1	1	1
C	1	1	1	1
D	0	0	0	1