

Logics, Categories, and Colimits for Artificial Intelligence

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Exercise Sheet 6

Due: December 5, 2008

Exercise 6.1 (Formalization in First-Order Logic)

Translate the following into First-Order Logic. Explain the meanings of the names, predicates, and functions you use, and comment on any shortcomings in your translations.

- (a) There's a sucker born every minute.
- (b) Whither thou goest, I will go.
- (c) Soothsayers make a better living in the world than truthsayers.
- (d) To whom nothing is given, nothing can be required.
- (e) If you always do right, you will gratify some people and astonish the rest.

Exercise 6.2 (First-Order Theorem Proving)

Use SPASS to prove that the following statements are tautologies:

- (a) There is some x such that if x is a P then everything is a P .
- (b) There cannot be any barber (who is a man) who shaves every man who does not shave himself.

Exercise 6.3 (Satisfiability of Specifications)

Consider the specification of classical extensional parthood¹. Assume you want to prove that this specification is consistent (right click on the node, node menu, check consistency, CASL \rightarrow SoftFOL (darwin)).

- (a) Install the latest HETS binary², the DARWIN THEOREM PROVER³ and the E EQUATIONAL THEOREM PROVER⁴.
- (b) Simplify the task of checking consistency by decomposing the specification in such a way that the whole specification can be seen as a conservative extension of a smaller specification and it is sufficient to prove consistency of the smaller specification and conservativity of the extension.
- (c) Use DARWIN to find some models for the specification (say the three smallest models). For the smallest model, simply run DARWIN from HETS as described above. To find larger models of size n , add to your specification axioms stating that there should be n pairwise distinct elements in the model. Can you come up with a conjecture regarding the general form of a model for CLASSICAL_EXTENSIONAL_PARTHOOD? How many elements can a model have?

¹<http://www.informatik.uni-freiburg.de/~ki/teaching/ws0809/aillogic/CEP.casl>

²<http://www.dfki.de/sks/hets/>

³<http://combination.cs.uiowa.edu/Darwin/>

⁴<http://www.e prover.org/>

Exercise 6.4 (Induction Proofs)

Consider the definitions of natural numbers, lists, and trees in the file `Datatypes.casl`⁵. Use ISABELLE⁶ or SPASS⁷ to prove the implied sentences.

The exercise sheets may and should be worked on in groups of two (2) students. Please write both names on your solution.

⁵<http://www.informatik.uni-freiburg.de/~ki/teaching/ws0809/ailogic/Datatypes.casl>

⁶<http://isabelle.in.tum.de/>

⁷<http://www.spass-prover.org/>