

Logics, Categories, and Colimits for Artificial Intelligence

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Exercise Sheet 3

Due: November 14, 2008

Exercise 3.1 (Specification extensions)

Construct a specification extension such that the basis specification has models with 0, 1, 2 and 3 different expansions.

Exercise 3.2 (Conservative extensions)

Consider the following specifications.

(i)

```
logic PROPOSITIONAL
spec BLOCKSHAPES =
  props cube tetrahedon
  • cube  $\vee$  tetrahedon
  •  $\neg$ (cube  $\wedge$  tetrahedon)
then                                     %%cons?
  prop dodecahedron
  • cube  $\vee$  tetrahedon  $\vee$  dodecahedron
  •  $\neg$ (cube  $\wedge$  dodecahedron)
  •  $\neg$ (tetrahedon  $\wedge$  dodecahedron)
  •  $\neg$ cube  $\Rightarrow$  dodecahedron
end
```

(ii)

```
logic PROPOSITIONAL
spec IMPLICATIONS =
  props a b c
  •  $a \Rightarrow b$ 
  •  $b \Rightarrow c$ 
then                                     %%cons?
  prop d
  •  $\neg(d \Rightarrow a)$ 
end
```

- Decide whether the extensions in (i) and (ii) are conservative. If they are not, provide models that cannot be extended.
- In case of lacking conservativity, use the theorem from the lecture to construct from the model a sentence that can be proven in the extended theory, but not in the base theory.

Exercise 3.3 (Pizza ontology)

Formalize the following statements from the pizza ontology in Hets:¹

- Pizza is food.
- Pizza base is food.
- Pizza topping is food.
- Pizzas, pizza bases, and pizza toppings are disjoint sets of things.
- A fish topping is a pizza topping.

¹The input format follows the Manchester syntax. To get an idea of what such a formalization might look like, you can review the formalization of the family ontology from the lecture at <http://www.informatik.uni-freiburg.de/~ki/teaching/ws0809/lccai/family.het>.

- A meat topping is a pizza topping.
- Pizzas have pizza toppings.
- Pizzas have unique pizza bases.
- Thin and crispy pizza bases are pizza bases.
- A thin and crispy pizza is a pizza that only has a thin and crispy base.
- An interesting pizza is a pizza that has at least three toppings.
- A vegetarian pizza is a pizza which has neither a meat topping nor a fish topping.

Exercise 3.4 (Deductive ontology)

Download and read the document describing the deductive ontology introduced in the lecture.²

Formalize the part of the deductive ontology describing the concepts

- Satisfiable,
- Theorem,
- WeakerTheorem,
- Equivalent,
- TautologousConclusion,
- EquivalentTheorem,
- Tautology,
- ContradictoryAxioms,
- SatisfiableConclusionContradictoryAxioms,
- TautologousConclusionContradictoryAxioms, and
- NoConclusion,

i.e., the left half of the graphic depicting the deductive ontology, using Manchester syntax. Follow these steps:

- Introduce basis concepts describing the status of the axioms (valid, satisfiable, unsatisfiable), the status of the conjecture (valid, satisfiable, unsatisfiable), and the possible entailment relations between the axioms and the conjecture (all models of the axioms are models of the conjecture, some models of the axioms are models of the conjecture, etc).
- For these basis concepts, formalize all subsumption, equivalence and disjointness relations that you are aware of.
- Define the eleven concepts listed above as intersections of (complements of) basis concepts. Follow the definitions of the concepts given in the Section *Deductive Statuses* of the document describing the deductive ontology.

The exercise sheets may and should be worked on in groups of two (2) students. Please write both names on your solution.

²<http://www.cs.miami.edu/~tptp/cgi-bin/SeeTPTP?Category=Documents&File=SZSOntology>