Logics, Categories, and Colimits for Artificial Intelligence

T. Mossakowski — University of Freiburg
Winter Semester 2008/2009 — Department of Computer Science

Exercise Sheet 2

Due: November 7, 2008

Exercise 2.1 (Formal Proofs)

(a) Evaluate the validity of the following argument. If it is valid, use the program *Fitch* to construct a formal proof to show this. Otherwise, use *Tarski's World* to construct a counterexample.[1]

$$
\begin{array}{r|l}
1 & \mathsf{Cube(a)} \vee (\mathsf{Cube(b)} \to \mathsf{Tet(c)}) \\
2 & \mathsf{Tet(c)} \to \mathsf{Small(c)} \\
3 & (\mathsf{Cube(b)} \to \mathsf{Small(c)}) \to \mathsf{Small(b)} \\
\hline
4 & \neg\mathsf{Cube(a)} \to \mathsf{Small(b)}
\end{array}
$$

(b) Consider the set $\mathcal{T} = \{(A \wedge B) \to \neg A, C \vee A, \neg A \to A, B\}$. Use *Fitch* to construct a formal proof showing that $\mathcal{T} \vdash \perp$.

(c) Consider the following truth table for the ternary connective $\diamondsuit$.

P	Q	R	$\diamondsuit(P, Q, R)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Express $\diamondsuit$ using only the connectives $\vee$, $\wedge$, and $\neg$. Can you simplify the result such that the simplified sentence has no more than two occurrences each of P, Q, and R, and no more than six occurrences of the Boolean connectives $\vee$, $\wedge$, and $\neg$?

Exercise 2.2 (Conservative Extensions)

[1] The programs can be downloaded from the lecture wiki (Resources/Software).

Consider your solution to Exercise 1.3 from the last exercise sheet, and consider the theory morphism $\sigma : (\Sigma_1, \Gamma_1) \to (\Sigma_2, \Gamma_2)$, where

$$\begin{aligned}
\Sigma_1 &= \{\mathit{black_exhaust}, \mathit{blue_exhaust}, \mathit{low_power}, \mathit{overheat}, \mathit{ping}, \\
&\quad \mathit{incorrect_timing}, \mathit{clogged_filter}, \mathit{low_compression}, \mathit{carbon_deposits}, \\
&\quad \mathit{clogged_radiator}, \mathit{defective_carburetor}, \mathit{worn_rings}, \mathit{worn_seals}\}, \\
\Sigma_2 &= \Sigma_1 \cup \{\mathit{replace_auxiliary}, \mathit{repair_engine}, \mathit{replace_engine}\},
\end{aligned}$$

Γ_1 contains all the axioms corresponding to the symptoms (the overheating engine and the fact that the ignition timing is correct) as well as all the axioms describing diagnostic rules (i.e., the formalizations of facts (i) through (vi) in the informal description in Exercise 1.3). Γ_2 contains all axioms from Γ_1 plus the three rules corresponding to facts (vii) through (ix). The morphism σ is the inclusion mapping from Σ_1 into Σ_2 mapping each proposition to itself.

(a) Show that σ is a model-theoretically conservative theory morphism.

(b) Reformulate your HETS specification such that (Σ_2, Γ_2) is specified as an extension to (Σ_1, Γ_1) using the **then** keyword. Additionally, indicate that the extension is supposed to be conservative using **%cons**. Use HETS to prove that this is indeed the case (you will need the latest nightly build of HETS to do that[2]).

Exercise 2.3 (Description Logics)

Note: This exercise is not graded.
Familiarize yourself with the *pizza ontology*.
It can be found at `http://www.co-ode.org/ontologies/pizza/`.

The exercise sheets may and should be worked on in groups of two (2) students. Please write both names on your solution.

[2] You can download the new HETS library from the lecture wiki (Resources/Software) and follow the installation instructions provided there.