Principles of AI Planning

Expressive power

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Motivation: Why Analyzing the Expressive Power?

- Expressive power is the motivation for designing new planning languages
- Often there is the question: *Syntactic sugar* or *essential feature*?

⇝ *Compiling away* or change planning algorithm?

→ If a feature can be compiled away, then it is apparently only *syntactic sugar*.

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Example: DNF Preconditions

- Assume we have **DNF preconditions** in STRIPS operators
- This can be **compiled away** as follows
- Split each operator with a DNF precondition $c_1 \lor \ldots \lor c_n$ into n operators with the same effects and c_i as preconditions

⇝ If there exists a plan for the original planning task there is one for the new planning task and *vice versa*

→ The planning task has almost the same size

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- Can we compile away **conditional effects** to STRIPS?
- Example operator: $\langle a, b \rhd d \wedge \neg c \rhd e \rangle$
- Can be translated into four operators: $\langle a \wedge b \wedge c, d \rangle, \langle a \wedge b \wedge \neg c, d \wedge e \rangle, \ldots$
- Plan existence and plan size are identical
- Exponential blowup of domain description!

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STRIPS_N : plain strips with negative literals
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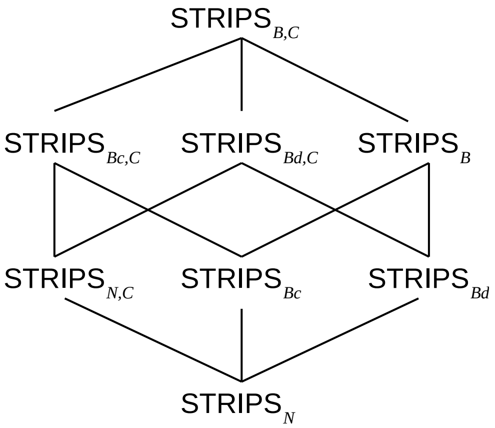
Ordering Planning Formalisms Partially



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Computational Complexity ...

Theorem

PLANEX is PSPACE-complete for $STRIPS_N$, $STRIPS_{C,B}$, and for all formalisms "between" the two.

Proof.

Follows from theorems proved in the previous lecture. □

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Measuring Expressive Power

Consider **mappings** between planning problems in different formalisms

- that **preserve**
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan “structure”
  - the solutions/plans themselves
- that **are limited**
  - in the *size* of the result (poly. size)
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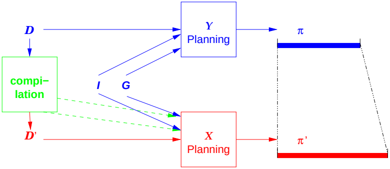
- Transform **domain structure** $\mathcal{D} = \langle A, O \rangle$ (with polynomial blowup) to $\mathcal{D}'$ preserving solution existence
- Only trivial changes to **states** (independent of operator set)
- Resulting **plans** π' should not grow too much (additive constant, linear growth, polynomial growth)

⇝ Similar to **knowledge compilation**, with operators as the *fixed part* and initial states & goals as the *varying part*



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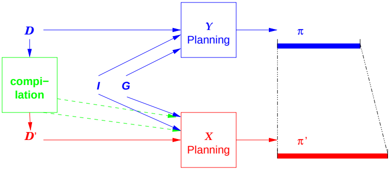

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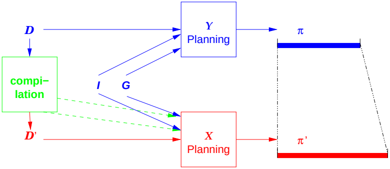

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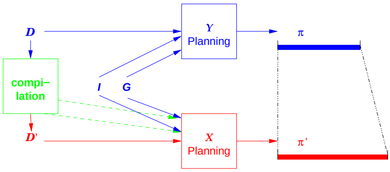

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- $\mathcal{Y} \preceq^c \mathcal{X}$: preserving plan size **linearly** (in $|\pi|$)
- $\mathcal{Y} \preceq^p \mathcal{X}$: preserving plan size **polynomially** (in $|\pi|$ and $|\mathcal{D}|$)
- $\mathcal{Y} \preceq^x_p \mathcal{X}$: **polynomial-time** compilability

Theorem

For all x, y, the relations $\preceq^x_y$ are transitive and reflexive.

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- $\mathcal{Y} \preceq^p \mathcal{X}$: preserving plan size **polynomially** (in $|\pi|$ and $|\mathcal{D}|$)
- $\mathcal{Y} \preceq^x_p \mathcal{X}$: **polynomial-time** compilability

Theorem

For all x, y, the relations $\preceq^x_y$ are transitive and reflexive.

Compilability

$\mathcal{Y} \preceq \mathcal{X}$ ($\mathcal{Y}$ is compilable to $\mathcal{X}$)
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Back-Translatability

- Shouldn't we also require that plans in the compiled instance can be *translated back* to the original formalism?
- Yes, if we want to use this technique, one should require that!
- In all *positive cases*, there was never any problem to translate the plan back
- For the *negative case*, it is easier to prove **non-existence**
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A (Trivial) Positive Result: STRIPS$_{Bd}$ $\preceq_p^1$ STRIPS$_N$

DNF preconditions can be **"compiled away."**

Assume operator $o = \langle c, e \rangle$ and

$$c = L_1 \vee \ldots \vee L_k$$

with L_i being a conjunction of literals. Create k operators $o_i = \langle L_i, e \rangle$

1. compilation is solution-preserving,
2. $\mathcal{D}'$ is only polynomially larger than $\mathcal{D}$,
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Another Positive Result: $\mathsf{STRIPS}_{C,Bc} \preceq_p^c \mathsf{STRIPS}_{C,N}$

CNF preconditions can be **"compiled away"** – provided we have already conditional effects.

- Evaluate the truth value of all disjunctions appearing in operators by using a **special evaluation operator** with conditional effects that make new "clause atoms" true
- Alternate between executing original operators (clauses replaced by new atoms) and evaluation operators

⇝ Operator sets grow only **polynomially**

⇝ Plans are **double as long** as the original plans

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A First Negative Result: Conditional Effects Cannot be Compiled into Boolean Preconditions

Consider domain $\mathcal{D}$ with only one ($\text{STRIPS}_{C,B}$) operator o:

$$\langle \top, (p_1 \rhd \neg p_1) \wedge (\neg p_1 \rhd p_1) \wedge \ldots \wedge (p_k \rhd \neg p_k) \wedge (\neg p_k \rhd p_k) \rangle,$$

which "inverts" a given state. For all (I, G) with

$$G = \bigwedge \{ \neg v \mid v \in A, I \models v \} \wedge \bigwedge \{ v \mid v \in A, I \not\models v \},$$

there exists a $\text{STRIPS}_{C,B}$ one-step plan.

Assume there exists a compilation preserving plan size linearly leading to a STRIPS_B domain structure $\mathcal{D}'$. There are exponentially many possible initial states, but only polynomially many different c-step plans for $\mathcal{D}'$. Some STRIPS_B plan π is used for different initial states I_1, I_2 (for large enough k). Let v be a variable with $I_1(v) \neq I_2(v)$.

$\rightsquigarrow$ In one case, v must be set by π, in the other case, it must be cleared.

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k-**FISEX**: Planning problem with fixed plan length k and varying initial state. Does there exist an initial state leading to a successful k-step plan?
1-FISEX is NP-complete for STRIPS$_{Bc}$ (= SAT).
k-FISEX is polynomial for STRIPS$_{N}$ (regression analysis)

$\rightsquigarrow$ STRIPS$_{Bc}$ $\not\preceq_{p}^{c}$ STRIPS$_{N}$ (if P$\neq$NP)

Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses **non-uniform complexity classes** such as *P/poly*.

- $\rightsquigarrow$ **Bäckström's conjecture holds** in the compilation framework.

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A Final Negative Result: Boolean Preconditions Cannot be Compiled Away Even in the Presence of Conditional Effects

- Boolean preconditions have the power of **families of Boolean circuits with logarithmic depth** (because Boolean formula have this power) ($= \mathrm{NC}^1$)
- Conditional effects can simulate only **families of circuits with fixed depth** ($= \mathrm{AC}^0$).
- The parity function can be expressed in the first framework (NC^1) while it cannot be expressed in the second (AC^0).

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- Boolean preconditions have the power of **families of Boolean circuits with logarithmic depth** (because Boolean formula have this power) ($= \mathsf{NC}^1$)
- Conditional effects can simulate only **families of circuits with fixed depth** ($= \mathsf{AC}^0$).
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Boolean Circuits

- We know what Boolean circuits are (directed, acyclic graphs with different types of nodes: *and, or, not, input, output*)
- **Size of circuit** = number of gates
- **Depth of circuit** = length of longest path from input gate to output gate
- When we want to *recognize formal languages* with circuits, we need a *sequence of circuits* with an increasing number of input gates $\rightsquigarrow$ **family of circuits**
- Families with polynomial size and poly-log ($\log^k n$) depth
- complexity classes NC^k (Nick's class)
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The classes AC^k

- The classes NC^k are defined with a fixed fan-in
- If we have *unbounded fan-in*, we get the classes AC^k
  - gate types: NOT, n-ary AND, n-ary OR for all $n \geq 2$
- Obviously: $\mathsf{NC}^k \subseteq \mathsf{AC}^k$
- Possible to show: $\mathsf{AC}^{k-1} \subseteq \mathsf{NC}^k$
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Accepting languages with families of domain structures with fixed goals

- We will view *families of domain structures* with fixed goals and fixed size plans as "machines" that accept languages
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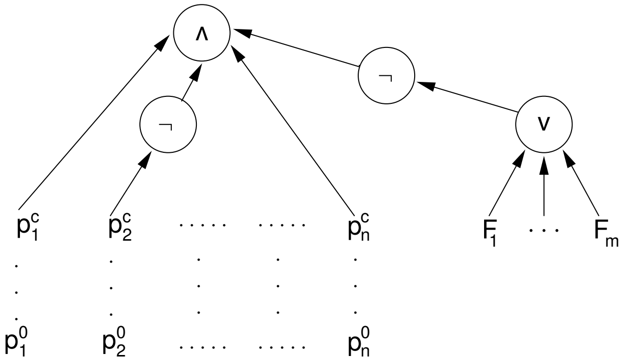
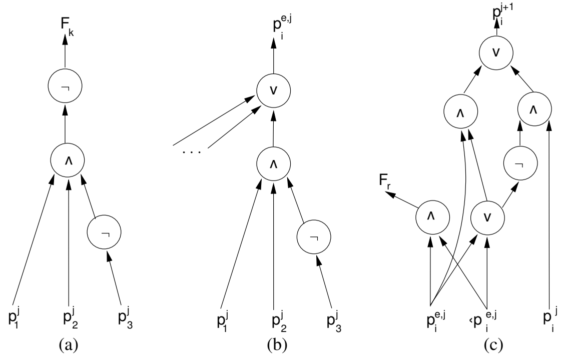
Simulating STRIPS$_{C,N}$ c-Step Plans with AC^0 circuits (1)

- Represent each operator and then chain the actions together ($O(|O|^c)$ different plans):



Simulating STRIPS$_{C,N}$ c-Step Plans with AC^0 circuits (2)

- For each single action (precondition testing (a), conditional effects (b), and the computation of effects (c)



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Proof.

Assuming $\mathsf{STRIPS}_B \preceq^c \mathsf{STRIPS}_{C,N}$ has the consequence that the underlying compilation scheme could be used to compile a NC^1 circuit family into an AC^0 circuit family, which is impossible in the general case. □

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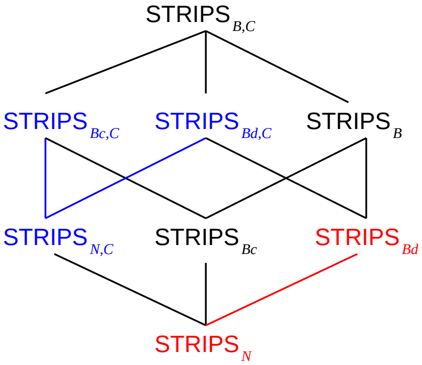
General Results for Compilability Preserving Plan Size Linearly



All other potential positive results have been ruled out by our 3 negative results and transitivity.

Summary

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→ **Results are relevant for building planning systems**

⇝ **CNF preconditions** do not add much when we have already conditional effects

- Note: In all cases we can get a positive result if we allow for a polynomial blow-up of the plans.

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