

Advanced AI Techniques

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WS 2006/2007

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Exercise Sheet 13

Due: Tuesday, 13. Februar 2007

Exercise 13.1 (Weakly Dominated Strategies)

A *Second Price Sealed Bid Auction* (Vickrey auction) is an auction where all bidders simultaneously (equivalent: secretly) submit bids so that no bidder knows the bid of any other participant. The winning bidder has to pay the second highest bid.

This can be formalized as follows: $\langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ with:

- $N = \{1, \dots, n\}$ where $n \geq 2$. Players i evaluation of an object is $v_i \in \mathbb{R}$ and $v_1 > v_2 > \dots > v_n > 0$ holds.
- For all $i \in N$ is $A_i = \mathbb{R}^+$, where $a_i \in A_i$ is a bid of a_i Euro.
- The utility function u_i is given as follows: If player i wins then $u_i(a) = v_i - \max a_{-i}$. The player gets the object (utility v_i), but has to pay the highest price of the other players ($\max a_{-i}$). If player i does not win, then $u_i(a) = 0$.

The object is given to the player with the lowest index among those who submit the highest bid in exchange for the payment.

Show that for each parameter v_i there is a weakly dominated strategy, i.e. a strategy that weakly dominates all alternatives. Show also that this weakly dominated strategy is a Nash equilibrium.

Exercise 13.2 (Iterative Elimination and Nash Equilibrium)

Show: If the method of iterative elimination solves a strategic game uniquely, the resulting strategy profil is a *Nash equilibrium* and it is *unique*.

Exercise 13.3 (Nash Equilibrium in Mixed Strategies)

Take the game „Rock – Scissor – Paper” from Exercise 11.

- Determine the Nash equilibrium in mixed strategies for this game.
- Show that there are no other Nash equilibria in mixed strategies for this game.

Please hand in a joint solution of three students and write all names on the sheet.