Semantic Networks and Description Logics

Description Logics – Implementation Techniques

Knowledge Representation and Reasoning

January 9, 2005

Description Logics – Implementation Techniques

Motivation

Classification

Subsumption Tests

Provisional Balance

Tableau Proofs

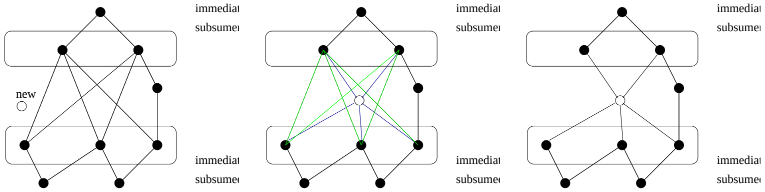
Preprocessing

Motivation

- It is easy to write a subsumption checker for, e.g., $\mathcal{ALC}$
- It is also easy to extend this to a small TBox/Abox system
- On top of that we need
  1. a graphical user interface
  2. an application interface with the right kind of services
  3. an **efficient implementation** of the basic reasoning services
- Possible areas of optimization:
  1. Classification
  2. Subsumption test
  3. Preprocessing the KB

Classification: Sorting in Partial Orders

- Compute the hierarchy induced by the subsumption relationship (similar to sorting)
- Also important in other contexts
- Incremental creation
  1. Find immediate subsumers and subsumees in hierarchy
  2. Place new concept at this place (and delete superfluous links)



Worst-Case Lower Bound for Classification

- Count number of subsumption tests
- What is the worst case?
- Worst case: All concepts are *unrelated*
- $O(n^2)$ for n being the number of concept names.
- Analytical comparison (for the *average case*) between different methods is probably difficult because it is unknown, how many partial orders exist for a given number of elements
- Empirical comparisons are the best we can hope for

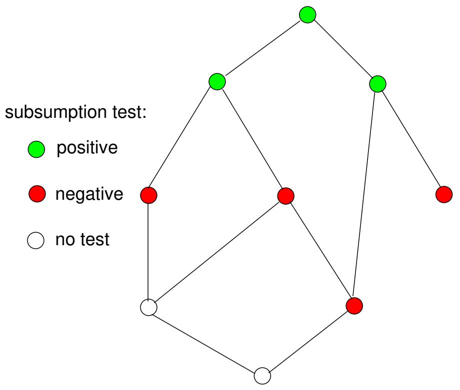
Method 0: Brute force

The brute-force method. For each new concept C:

- For each concept D in the hierarchy, check
  - whether C subsumes D and
  - whether D subsumes C.
- Compute from those the immediate subsumers and subsumees
- $2n$ tests for each new concept
- $\sum_{i=0}^{n-1} 2i = n \times (n-1)$ for the whole KB

Method 1: Simple Traversal

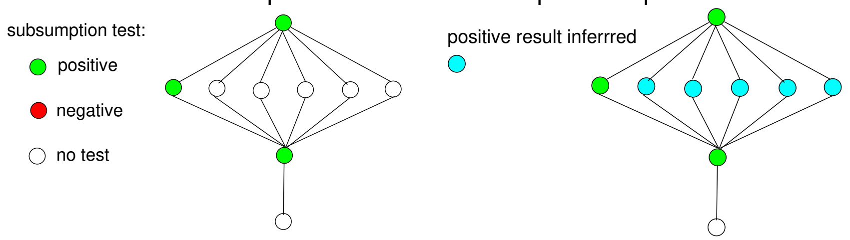
- **Top search**: Traverse hierarchy top down (e.g. depth-first), and try to identify direct subsumers by checking subsumption and mark subsumption failures.



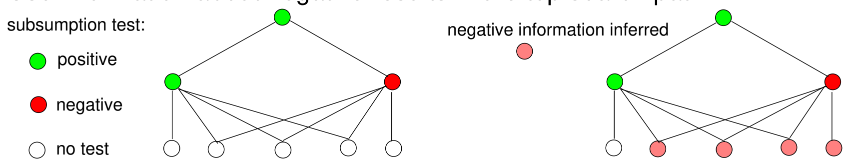
- **Bottom search**: dual, from bottom element

Method 2: Smart Traversal

- Exploit information from previous subsumption tests
- Use information about positive results in the top search path



- Use information about negative results in the top search path

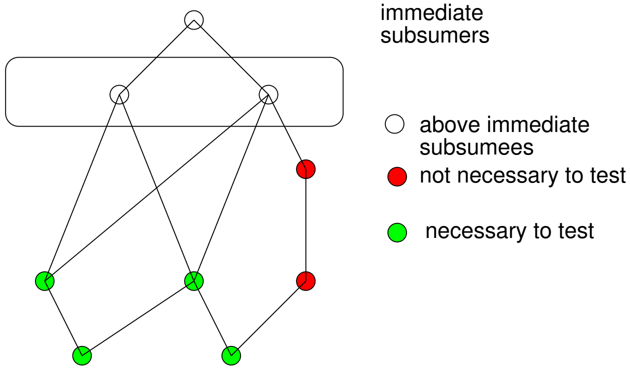


Three Alternatives

- *Note:* Smart traversal is provably as good as simple traversal (counting the number of subsumption tests) and the additional overhead is negligible (0.5% of the subsumption costs in a typical KB)
- Alternatives:
  1. Depth-first search using only positive information
  2. Breadth-first search using only negative information
  3. Depth-first search using both negative and positive information
- Our results: Breadth-first search is best. Using only positive information does not help much.

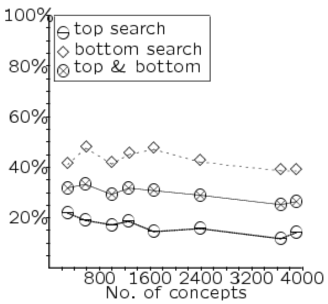
Smart Traversal: Bottom Search

- Dual optimization to top search (starting at bottom)
- In addition, it is sufficient to consider only nodes that are subsumed by all immediate subsumers!

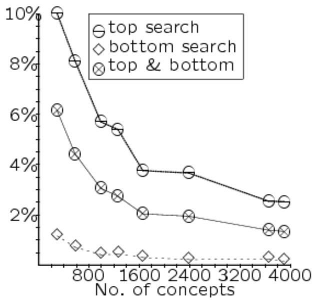


Number of Subsumption Tests Relative to "Brute Force" Method

- In 1992, we evaluated the effects of different techniques
- On real KBs, the book keeping overhead turned out to be relatively low compared with the time for subsumption checking
- We measured the effects on randomly generated KBs modeled after the structure of real KBs



Simple traversal



Smart traversal

Method 3: Insertion into Chains

- Building up a linearly ordered list incrementally, it is a good idea to use binary search for identifying the next insertion point
- Idea:
  - Compute a good **chain-covering** of the partial order, i.e., a set of linearly ordered chains covering all elements
  - Minimal chain covering is (of course) NP-hard. Therefore we used a heuristic
  - Do a binary search in all chains
  - and apply the same propagation of positive and negative information as in the smart traversal
  - One gets a super-set of the set of immediate subsumers (and subsumees)
  - in most typical cases a little bit worse than with smart traversal (approx. 10%)
  - better only if the connectedness of the order was higher than in typical KBs

Results for Chain Insertion

Type of KB	No. of nodes	Average degree	Average no. of pred. & succ.	Breadth	Depth	Relative no. of comparisons
	184	1.71	5.67	105	6	103.7%
	241	1.91	6.38	124	6	100.3%
real	254	1.99	13.02	135	6	91.8%
KB	269	1.72	5.16	164	7	107.9%
	330	1.85	8.13	141	12	110.0%
	298	2.36	8.88	142	8	115.7%
random	583	2.58	12.24	330	7	114.5%
KB	992	2.73	16.77	478	10	111.7%
mod.	1263	3.18	16.61	661	11	108.9%
after	1659	3.19	18.86	927	10	110.3%
real	2389	3.50	25.49	1188	10	111.3%
KB	3658	3.82	27.20	1703	8	105.3%
	3905	4.04	33.95	1858	11	99.9%
	301	7.67	42.11	88	8	73.2%
	301	6.40	10.43	168	6	102.7%
	301	4.22	5.68	205	4	101.3%
	586	9.93	72.55	144	9	67.2%
	586	10.39	20.42	301	7	103.3%
random	586	7.72	11.50	353	5	102.9%
partial	995	5.52	250.24	85	28	16.2%
orders	995	16.40	62.88	354	9	91.4%
	995	13.58	28.38	506	6	105.1%
	1266	5.78	321.19	100	30	12.9%
	1266	18.22	76.87	438	9	89.8%
	1266	17.82	41.70	592	6	101.1%

Exploiting the Concept Structure

1. If a concept is defined conjunctively, i.e.,

$$A \doteq X \sqcap Y \sqcap \ldots \sqcap Z$$

   all atomic concepts (X, Y, Z) are obviously super-concepts. So one can mark these concepts as subsuming (and can propagate this information)
2. Primitive concepts (introduced with $\sqsubseteq$) can be ignored in the bottom search phase, if we classify in definition order (i.e., before a term is used, it will be defined)
3. Primitive components of a concept are the undefined concepts in the unfolded form of a concept. A subsumption test between two conjunctively defined concepts can only be successful if the primitive components are in an set-inclusion relationship

Caching and Lazy Unfolding

- Instead of unfolding a concept and then testing subsumption, one can delay the expansion of a defined concept until no other way to prove unsatisfiability is possible
- Instead of testing for clashes of primitive concepts, we also look for clashes on the level of named, defined concepts.
- Finally, we can in addition also look for clashes implied by the subsumption hierarchy, e.g., `Man` and `¬Human` form a clash.
- These optimization saved roughly 50% of the runtime on the realistic KBs

Provisional Balance

- The described methods (and a few others) were used in speeding up the KRIS system
- We achieved a speedup factor of around 400
- The difference between a first prototype implementation that just implements the principles and a highly optimized system can be huge – in particular when we are dealing with worst-case exponential time/space problems
- Interestingly, the method we believed to be superior (chain insertion) was not effective at all
- One has to look for what *typical cases* require – and these cases might not be easily recognizable as *polynomial special cases*
- There are techniques especially geared towards more expressive DLs – and some we will have a short look at

Semantic Branching in Tableau Proofs

- Consider the System: $S = \{x : (A \sqcup B), x : (A \sqcup C)\}$
- Ordinarily,
  - we would first try out A and B, nondeterministically.
  - Then we would make another nondeterministic choice between A and C.
- if A leads to clash, we would try that out twice.
- Use *semantic branching* instead (similar to what is done in the Davis-Putnam procedure)
- Branch over A and $\neg A$!
- This avoids double work on A and leads to tighter constraints (when using $\neg A$ in the other branch)
- Experience shows that the additional overhead for propagating $\neg A$ is usually negligible
- *Local simplification* becomes possible

Boolean Constraint Propagation

- When using semantic branching, we can also simplify disjunctions by propagating the concepts, we have branched on, using *Boolean constraint propagation* (aka *unit propagation*).
- Example:

$$S = \{x : (C \sqcup (D_1 \sqcap D_2)), x : (\neg D_1 \sqcup \neg D_2 \sqcup C), x : \neg C\}$$

- Because of $x : \neg C$, this can be simplified to

$$S = \{x : (D_1 \sqcap D_2), x : (\neg D_1 \sqcup \neg D_2 \sqcup C), x : \neg C\}$$

- This in turn can be simplified to

$$S = \{x : (D_1 \sqcap D_2), x : C, x : \neg C\}$$

- This is easily be detected as unsatisfiable!
- Note that no nondeterministic branching was necessary!

More Methods for Speeding up Tableau Proofs

- *Dependency-directed backtracking*: Try to detect the reason for a clash and jump back (instead of exploring fruitless alternatives)
- *Heuristic guided search*: Branch on the right disjunct
- *MOMS heuristic*: Branch on the disjunct with the **m**aximum number of **o**ccurrences in disjunctions of **m**inimum **s**ize.
- Seems not to be a good heuristic in DLs.
- The best heuristic seems to be *oldest first*, which selects a disjunct from the disjunction depending on the least recent branching point
- Other techniques such as caching tableau trees and reusing them are also possible

Avoiding Internalization

- Internalization is a powerful technique to deal with general concept inclusion statements (we have seen in the last lecture)
- However, it introduces a lot of *non-deterministic choices*. Remember: $C_i \sqsubseteq D_i$ is translated into $D_i \sqcup \neg C_i$, which is enforced on every domain element (using the quasi-universal role)
- Each inclusion statement leads to a nondeterministic choice point for each element!
- Avoid if possible:
  - Collect multiple primitive definitions:

$$(A \sqsubseteq C_1), \ldots (A \sqsubseteq C_n) \Longleftrightarrow (A \sqsubseteq C_1 \sqcap \ldots \sqcap C_n)$$

  - Partition the terminology into two parts $\mathcal{T}_u$ and $\mathcal{T}_g$, where the former is unfoldable and the latter is the general part. Then we can apply *lazy unfolding* to all statements in $\mathcal{T}_u$.

Absorption

- Since general inclusion statements lead to nondeterminism, it is a good idea to reduce these as much as possible.
- Use the following equivalences to *absorb* general inclusion statements into primitive concept definitions:

$$C_1 \sqcap C_2 \sqsubseteq D \iff C_1 \sqsubseteq D \sqcap \neg C_2$$
$$C \sqsubseteq D_1 \sqcap D_2 \iff (C \sqsubseteq D_1), (C \sqsubseteq D_2)$$

- Example Absorption. Assume: $\texttt{Geometric-figure} \sqsubseteq \texttt{Figure}$, $\texttt{Geometric-figure} \sqcap \exists \texttt{angles.Three} \sqsubseteq \exists \texttt{sides.Three}$.
- The latter inclusion statement can be massaged into:

$$\texttt{Geometric-figure} \sqsubseteq \exists \texttt{sides.Three} \sqcup \neg \exists \texttt{angles.Three}$$

- Then it can be absorbed into:

$$\texttt{Geometric-figure} \sqsubseteq \texttt{Figure} \sqcap \exists \texttt{sides.Three} \sqcup \neg \exists \texttt{angles.Three}$$

- Using this technique can make a huge difference

Conclusion

- Implementing inference methods for DLs is easy – if we do not care about *efficiency*
- The difference between a prototype implementation and a carefully optimized system can be several orders of magnitude
- Optimizations are possible on classification level, the subsumption testing level, and the tableau proof level
- These optimization should be geared towards the *typical case*
- It is nearly impossible to decide analytically, which method is the right one, only *empirical comparisons* can help here
- **Note**: Although there exists now a number of efficient DL methods, in general there will remain the problem that sometimes these systems just have to give up because a subsumption query is to difficult!

Literature

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- I. Horrocks, Implementation and Optimization Techniques, in: Baader, F., D. Calvanese, D. L. McGuinness, D. Nardi, and P. F. Patel-Schneider (eds.), *The Description Logic Handbook: Theory, Implementation, Applications*, Cambridge University Press, Cambridge, UK, 2003.