Advanced AI Techniques (WS05/06)

Exercise sheet 12 (Game Theory)
Deadline: Tuesday, 14 Feb 06

Exercise 1 (Game n-target-strike, Mixed Strategies, 4 points)

Solve the following task in terms of mixed strategies using the linear programming method (no other way to find a solution is given credits).

An army wants to attack an important strategic target of its enemy. It has exactly one missile to bomb one of the targets. There are three targets that might be worth attacking: 1, 2, and 3.
However, the enemy has a defense system. It can protect exactly one of the targets. Both decide without knowledge of the other army's action.
As usual in times of war, the attacking army wishes to maximize the expected value of the destroyed target, the defending army wants to minimize it. If the attacking army attacks an unprotected target, it is destroyed. If the attacked target is protected, nothing is destroyed and both get a payoff of 0 (weapons and shields are very cheap in this idealistic game). If target i is destroyed, the attacker gets a payoff of $+i$, the defender of $-i$.

a. Formulate the payoff matrix and check if there is a pure strategy NE.

b. Can there be a mixed strategy NE with a support of one or two targets?

c. Define the inequalities for the corresponding mixed strategy game in terms of linear programming. What is the function to be maximized?

d. Solve it with a free linear (in)equality solver, e. g. lp_solve.

Exercise 2 (4 points)

Mom and daughter want to buy a pet. There are four pets - a dog, and three little fury ones: a chinchilla, a hamster, and a guinea pig. Mom decides whether she will participate in taking care of the pet, or whether this will be the daughter's job. The daughter decides which pet she will take. But dad (who pays for it) wants that both tell him her decision independently.
They know that mom likes dogs, and would even prefer to take care of it (+4) rather

than letting her daughter take care of it alone (+3). If she has to take care of any other pet, the valuation for mom would be a burden (-3). If she does not need to take care about her daughter's pet, she'd be happy (+3) and it makes no difference which pet it is.
For the daughter, a dog is worth $+4$ if mom helps her caring for it, and only $+2$ if mom doesn't. Even more, she'd like to have a fury little animal, but she's yet undecided which one. Having her prefered pet is worth (+6) with mom's help, and (+3) without. However, having the "wrong" pet, doesn't give her any pleasure.

a. Formulate this game in terms of states for each prefered fury pet. Give the payoff matrix for each state.

b. Solve the game (find a NE) when the prefered state is not known (each state having equal probability of $\frac{1}{3}$).

c. Should the daughter make up her mind and tell mom about it? What would change? Resolve the game.

Exercise 3 (4 points)

As a variation of the previous game, assume that the daughter has made up her mind which pet she likes best, but hasn't told her mom yet. She can signal to her mom a preference by sighing when she sees the prefered fury little pet. But mom is not sure what a sigh means. Assume when she hears a sigh when the daughter sees the chincilla, the probability that the chincilla is the prefered fury little pet is p, and $\frac{1-p}{2}$ for hamster and guinea pig, each.
Assume mom decides after hearing a sigh for a chincilla. Does sighing help little daughter?

a. Formulate this as a game with signals, and define the beliefs.

b. Is there always a NE of pure strategies, or is there some p value where mixed strategies must be applied? Would this open a chance that the daughter gets her prefered pet, and mom takes care of it?