Advanced AI Techniques (WS05/06)

Exercise sheet 11 (Game Theory)
Deadline: Tuesday, 7 Feb 06

Exercise 1 (Kitchen Cleaning Game, 4 points)

Consider again last week's n person game: Kitchen Cleaning.

Let n people live in a common household (shared appartment?).The kitchen is untidy and dirty. Each person makes its own decision whether s/he cleans the kitchen. The "reward" for the decision to clean the kitchen is -1 (because it takes time and isn't fun). If at least one person decides to clean the kitchen, everybody profits from having a clean kitchen (+1 for everybody). For the person(s) who cleaned the kitchen, the negative and positive rewards are balanced out to a total payoff of 0. If nobody cleans the kitchen, everybody gets a payoff of 0 (we can just tolerate an untidy kitchen).

a. How many Nash equilibria of pure strategies does the n-person problem have, and how do they look like?
b. Change the payoff matrix so that only an action profile in which exactly one person cleans the kitchen is a Nash equilibrium.
c. Change the payoff matrix so that the action profile in which everybody cleans the kitchen is the only Nash equilibrium.

Exercise 2 (4 points)

Give a proof for the claim:

If there exists a unique solution from iterated elimination of strictly dominated strategies, then it is a Nash equilibrium.

Can it happen that the solution, i.e. action profile, is not unique?

Exercise 3 (Second price auction game, 4 points)

In an auction game, an object is to be assigned to a player in the set $\{1, \ldots n\}$ in exchange for payment. Player i's valuation of the object is v_i, and we assume $v_1 > v_2 > \ldots > 0$. The mechanism used to assign the obect is a sealed-bid auction: The players simultaneously submit bids (nonnegative numbers), and the winner is the person with the highest bid, s/he gets the object. (If several equal highest bids are made, the person with the lowest index number is the winner.) In our "second price

auction", the payment that the winner makes is the highest bid amoung those submitted by the players who do not win (so that if only one player submits the highest bid, then the price paid is the second highest bid).

Show that in a secound price auction the bid v_i of any player i is a weakly dominant action: player i's payoff when he bids v_i is at least as high as his payoff when he submits any other bid, regardless of the actions of the other players. Show that nevertheless there are "inefficient" euqilibria in which the winner is not player 1.