Game Theory

14. Poker

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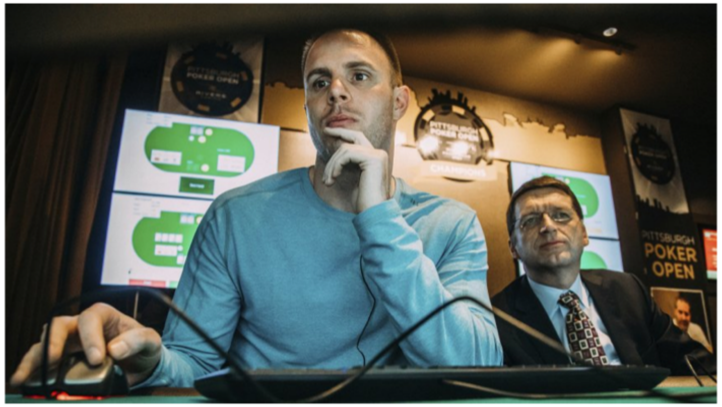
1 Motivation

Motivation

- The system Libratus played a Poker tournament (*heads up no-limit Texas hold 'em*) from January 11 to 31, 2017 against four world-class Poker players.
  - Heads up: One-on-One, i.e., a zero-sum game.
  - No-limit: There is no limit in betting, only the stack the user has.
  - Texas hold'em: Each player gets two private cards, then open cards are dealt: first three, then one, and finally another one.
  - One combines the best 5 cards.
  - Betting before the open cards are dealt and in the end: check, call, raise, or fold.
- Two teams (reversing the dealt cards).
- Libratus won the tournament with more than 1.7 Million US-$ (which neither the system nor the programming team got).

The humans behind the scene



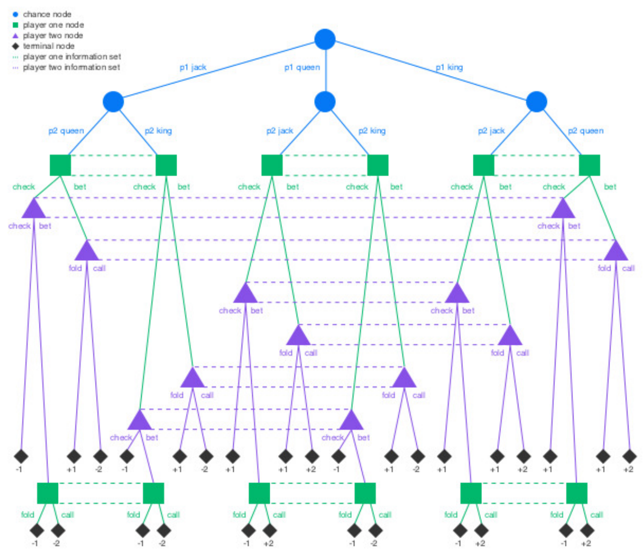
Professional player Jason Les and Prof. Tuomas Sandholm (CMU)

2 Kuhn Poker

Kuhn Poker

- Minimal form of heads-up Poker, with only three cards: Jack, Queen, King.
- Each player is dealt one card and antes 1 chip (forced bet in the beginning).
- Player 1 can check (declines to make a bet), or bet 1 chip.
- After player 1 has checked, player 2 can check or bet. If player 2 bets, player 1 can fold or call (also betting one chip)
- After Player 1 has bet, player 2 can fold or call.

Kuhn Poker: Game tree



Kuhn Poker: Results

Kuhn has shown:

- There exist a family of Nash equilibria behavioral strategies for player 1 and one behavioral NE strategy for player 2.
- In this Nash equilibrium, the expected payoff for player 1 is $-1/18$.
- That shows the systematic disadvantage, the first player has!

3 Real Poker: Problems and techniques

State space size

- Reminder: In chess, there are 10^{47} distinct states, in Backgammon there are 10^{20}.
- Heads-up limit Texas hold'em has 10^{17} distinct states and 10^{14} information sets.
- No-limit: Depends on stack. With 20k$: 10^{161} information sets.

General techniques

- Abstraction: Action abstraction (bet size) and card abstractions (classifying similar hands into buckets) $\rightarrow$ only 10^{12} information sets.
- Equilibrium computation: Using **counterfactual regret minimization** as a self-play technique.
- Sub-game solving: In later betting rounds, one solves the game with a finer abstraction (and the information gained from the game so far).
- Self-Improvement: During the night, new parts of the game tree are explored, when abstraction is too coarse there.
- 25 Million core hours to compute strategies.

4 Counterfactual regret minimization

Regret matching in strategic games

Play a strategic game for a number of rounds:

- Regret is determined after each game round: If I had played another move, my payoff would have been *that* much higher!
- Accumulate all positive regrets over time.
- Match the probabilities of a mixed strategy with the accumulated regret.

Take the average over all mixed strategies.

If two players use the regret matching technique in a zero-sum game, then the average over the mixed strategies converges to Nash equilibrium strategies.

Regret matching: RPS example with two rounds I

Assume we play rock, paper, scissors, and player 1 uses regret matching.

1. Initial cumulative regret is $(0,0,0)$.
2. If there is no positive accumulated regret, play uniform strategy $(1/3,1/3,1/3)$.
3. Player 1 chooses R, player 2 P.
4. Regret for player 1:
   - $R : u_1(R,P) - u_1(R,P) = -1 - -1 = 0$
   - $P : u_1(P,P) - u_1(R,P) = 0 - -1 = +1$
   - $S : u_1(S,P) - u_1(R,P) = 1 - -1 = +2$
5. Player 1's cumulative regret is now $(0,1,2)$
6. Regret matching suggests this strategy: $\alpha_1^1 = (0,1/3,2/3)$.
7. Player 1 chooses P, while player 2 chooses S

Regret matching: RPS example with two rounds II

8. Regret for player 1:
   - $R : u_1(R,S) - u_1(P,S) = 1 - -1 = +2$
   - $P : u_1(P,S) - u_1(P,S) = -1 - -1 = 0$
   - $S : u_1(S,S) - u_1(P,S) = 0 - -1 = +1$
9. Cumulative regret is now $(2,1,3)$
10. Regret matching: $\alpha_1^2 = (1/3,1/6,1/2)$
11. The average strategy is $(1/6,3/12,7/12)$. Well, not close to the NE strategy, but will converge!

Counterfactual regret minimization

- Regret matching in strategic games does not buy us anything. We know how to compute NEs for zero-sum games already.
- In extensive-form games, we can use it to modify our behavioral strategies at each information set.
- We have to “pass down” the probability that an information set is reached and have to “pass up” the utility of a terminal history.
- As in the strategic game case, the average strategy converges to a Nash equilibrium (in behavioral strategies).
- Is it good enough?
- Since a lot of histories are explored, also “off-NE strategies” will be visited and reasonable choice will occur.

Notation & Definitions I

- During training, t and T denote time steps.
- Let $\pi^\beta(h)$ be the probability that history h will be reached (depends on behavioral strategy profile β and chance moves).
- $\pi^\beta(I_i) = \sum_{h \in I_i} \pi^\beta(h)$ is then the probability that information set I_i will be reached.
- The counterfactual reach probability of I_i, written $\pi^\beta_{-i}(I_i)$, is the probability of reaching I_i under the assumption that player i always uses actions with probability 1 in order to reach I_i.
- If β is a behavioral strategy profile, then $\beta_{I_i \to a}$ is the same profile, except that at information set I_i, player i always plays a.

Notation & Definitions II

- If $z \in Z$ is a terminal history, then we write $h \sqsubset z$, if h is a proper prefix of z.
- For $h \sqsubset z$, the notation $\pi^\beta(h,z)$ is the probability that we reach z from h.
- The counterfactual utility of β at non-terminal history h is:

$$v_i(\beta,h) = \sum_{z \in Z, h \sqsubset z} \pi^\beta_{-i}(h)\pi^\beta(h,z)u_i(z).$$

- The counterfactual regret of not taking action a at history $h \in I_i$ is:

$$r(h,a) = v_i(\beta_{I_i \to a},h) - v_i(\beta,h).$$

Notation & Definitions III

- Counterfactual regret of not taking a at I_i:

$$r(I_i,a) = \sum_{h \in I_i} r(h,a).$$

- $r_i^t(I_i,a)$ refers to the regret in episode t, when players use β^t and i does not a in I_i.
- Cumulative counterfactual regret is then defined as:

$$R_i^T(I_i,a) = \sum_{t=1}^{T} r_i^t(I_i,a).$$

- Let us define the positive cumulative counterfactual regret as: $R_i^{T,+}(I_i,a) = max(R_i^T(I_i,a),0)$.

Notation & Definitions IV

- Now, the regret matching strategy for episode $T+1$ is called β^{T+1} and computed as:

$$\beta^{T+1}(I_i,a) = \begin{cases} \frac{R_i^{T,+}(I_i,a)}{\sum_{a \in A(I_i)} R_i^{T,+}(I_i,a)} & \text{if } \sum_{a \in A(I_i)} R_i^{T,+}(I_i,a) > 0 \\ \frac{1}{A(I_i)} & \textit{otherwise}. \end{cases}$$

CFR in action

- One use usually what is called chance sampling, i.e., one uses one or more shuffles of the cards to compute the values for one episode.
- That also means that only a small part of the game tree needs to be in main memory.
- After a fixed number of episodes one stops and then has an (approximate) NE.
- Although, we would have liked a sequential equilibrium, we most probably will also collect regret values for information set, which are not on equilibrium profile histories.
- There are many variations and refinements of CFR.
- Looks like reinforcement learning, but it is not.