Game Theory

8. Interlude: Applications

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1 Applications of Game Theory

Applications of Game Theory

- Wide range of applications of game theory
- Originally: in economics
- Now: ubiquitous, also in computer science and AI
  - robotics
  - cloud computing
  - social networks
  - resource management
  - ...

2 Security Games

- Motivation
- Setting
- Formalization
- Strategies and Payoffs
- Equilibria
- Theoretical Results

Security Games
Motivation

Today: Security games [Tambe et al., 2007ff.]

- infrastructure security games (air travel, ports, trains)
- green security games (fisheries, wildlife)
- opportunistic crime security games (urban crime)

Some video lectures by M. Tambe:

- `https://www.youtube.com/watch?v=whl5TO7sMa8` (Infrastructure security games, 3 mins)
- `https://www.youtube.com/watch?v=61yHC5c2c-E` (Green security games, 8 mins)
- `https://www.youtube.com/watch?v=D4sxZm8-NdM` (ICAPS 2017 invited talk, 1 hour)

Security Games
Motivation

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Summary

Common setting in security games:

- attacker and defender
- defender wants to protect targets using patrolling units
- defender chooses probability distribution over routes such that expected damage is minimized given that the probabilities can be observed by attacker

Security Games
Setting

Unobservable vs. observable defense probabilities:

- Unobservable: strategic game
- Observable: extensive game

Example (Security game payoff matrix)

Attacker

		c	d
Defender	a	1,1	3,0
	b	0,0	2,1

Unobservable defense probabilities (strategic game): Only NE is (a,c).

Security Games
Setting

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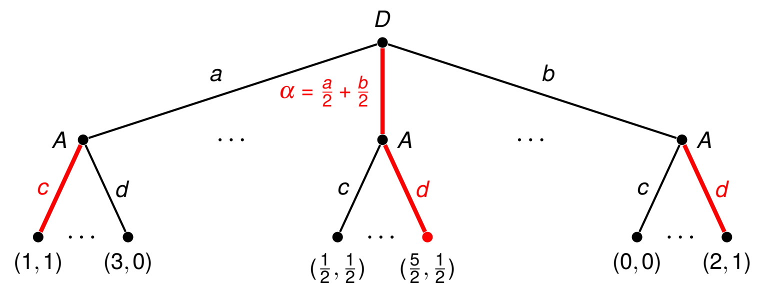
Example (Security game (ctd.))

Observable defense probabilities (extensive game, mixed strategies):



Subgame-perfect equilibrium (α, d).

Security Games
Formalization

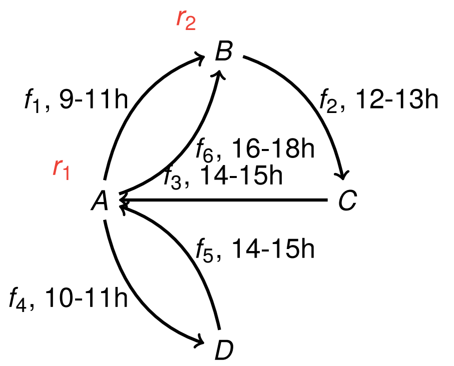
Definition (Security game)

A security game is a tuple $\langle T, R, (S_i), U_d^c, U_d^u, U_a^c, U_a^u \rangle$, where

- $T = \{t_1, \dots, t_n\}$ is a finite set of targets,
- $R = \{r_1, \dots, r_K\}$ is a finite set of resources,
- $S_i \subseteq 2^T$ is the set of schedules that r_i can cover. A schedule $s \in S_i$ is a set of targets that can be covered by r_i simultaneously.
- $U_y^x(t_i)$ is the utility of player $y \in \{\textbf{a}ttacker, \textbf{d}efender\}$, if target t_i is attacked and is $x \in \{\textbf{c}overed, \textbf{u}ncovered\}$.

Security Games
Formalization

Example (Federal air marshal service)



- $T = \{f_1, f_2, f_3, f_4, f_5, f_6\}$
- $R = \{r_1, r_2\}$
- $S_1 = \{\{f_1, f_2, f_3\}, \{f_4, f_5\}\}$
- $S_2 = \{\{f_2, f_3, f_6\}\}$
- $U_y^x(t_i)$ unspecified

Security Games
Strategies and Payoffs

- Attacker pure strategies: $A_a = T$
- Attacker mixed strategies: $\Delta(T)$
- Defender pure strategies: allocations of resources to schedules, i. e., $\bar{s} = (s_1, \dots, s_K) \in \prod_{j=1}^{K} S_j$.

  Target t_i is covered in $\bar{s}$ iff $t_i \in s_j$ for at least one j, $1 \le j \le K$. Allocation $\bar{s}$ induces coverage vector $\bar{d} = (d_1, \dots, d_n) \in \{0, 1\}^n$ with $d_i = 1$ iff t_i is covered in $\bar{s}$.

  Let $\mathscr{D}$ be the set of coverage vectors for which there is an allocation $\bar{s}$ inducing it.

Security Games
Strategies and Payoffs

- Defender mixed strategies: $\Delta(\mathscr{D})$. For $\alpha_d \in \Delta(\mathscr{D})$, let $c_i = \sum_{\bar{d}=(d_1,\dots,d_n) \in \mathscr{D}} d_i \cdot \alpha_d(\bar{d})$ be the covering probability of target t_i.

  Notation: $\phi(\alpha_d) = (c_1, \dots, c_n)$.

  Example: $\bar{d}_1 = (1, 1, 0)$, $\bar{d}_2 = (0, 1, 1)$, $\alpha_d(\bar{d}_1) = \alpha_d(\bar{d}_2) = \frac{1}{2}$. Then $(c_1, c_2, c_3) = (\frac{1}{2}, 1, \frac{1}{2})$.
- Payoffs: Let $(\alpha_d, \alpha_a) \in \Delta(\mathscr{D}) \times \Delta(T)$ be a mixed strategy profile. Expected utility of player $y \in \{a, d\}$:

$$U_y(\alpha_d, \alpha_a) = \sum_{i=1}^{n} \alpha_a(t_i) \cdot \left(c_i \cdot U_y^c(t_i) + (1 - c_i) \cdot U_y^u(t_i)\right).$$

Definition of best responses, Nash equilibria (NE) and maximinimizers (MM) as usual/expected. Hence omitted here.

More interesting scenario:

- Defender first commits to a mixed defense strategy.
- Attacker observes it over extended time period and learns probabilities.
- Attacker choses response $\alpha_a = g(\alpha_d)$ based on those observations. g is his response function.

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Definition (Strong Stackelberg equilibrium)

A pair $\langle \alpha_d, g \rangle$ is called a strong Stackelberg equilibrium (SSE) if the following holds:

- $U_d(\alpha_d, g(\alpha_d)) \geq U_d(\alpha'_d, g(\alpha'_d))$ for all α'_d;
- $U_a(\tilde{\alpha}_d, g(\tilde{\alpha}_d)) \geq U_a(\tilde{\alpha}_d, g'(\tilde{\alpha}_d))$ for all $\tilde{\alpha}_d$ and all g'; and
- tie breaking: $U_d(\tilde{\alpha}_d, g(\tilde{\alpha}_d)) \geq U_d(\tilde{\alpha}_d, \tau(\tilde{\alpha}_d))$ for all $\tilde{\alpha}_d$ and all $\tau(\tilde{\alpha}_d)$ that are attacker best responses to $\tilde{\alpha}_d$.

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Theorem

Defender NE strategies and defender MM strategies are the same. □

Theorem

NE strategies are interchangeable. □

Theorem

Defender SSE utilities are always at least as large as defender NE utilities. □

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Definition (Subsets of schedules are schedules property)

A security game satisfies the SSAS property ("subsets of schedules are schedules") if for all $r_i \in R$, for all $s \in S_i$, and for all $s' \subseteq s$, also $s' \in S_i$.

Remark: SSAS often "natural" to achieve, by "doing nothing".

Theorem

If SSAS holds, then every defender SSE strategy is also a defender NE strategy. □

Consequence: When choosing between SSE and NE strategies (assuming being observed or not), for the defender it is unproblematic to restrict attention to SSE strategies. NE interchangeability $\rightsquigarrow$ no risk of chosing a "wrong" NE strategy.

Outlook:

- With homogeneous resources and a small restriction on utility functions: then there exists unique defender MM strategy, which is also a unique SSE and NE strategy.
- Theory can be generalized to multiple attacker resources (attacking multiple targets simultaneously).

3 Summary

Summary

- Case study: security games (infrastructure, green, opportunistic crime)
- Modeled as Stackelberg games with strong Stackelberg equilibria (SSE)
- Results:
  - Though not zero-sum in general, similar results: defender NE = defender MM
    $\rightsquigarrow$ Nash equilibria interchangeable
    $\rightsquigarrow$ no equilibrium selection problem
  - Every defender SSE strategy also a NE strategy under reasonable assumption (SSAS)
    $\rightsquigarrow$ not knowing whether being observed is unproblematic