Game Theory

1. Introduction

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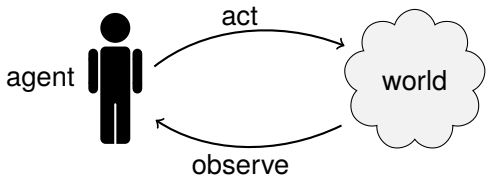
1 What is Game Theory?

Rational Agents

Consider rationally acting agents:

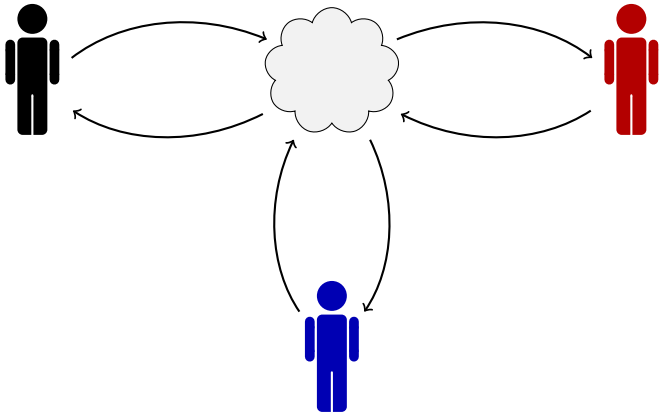


Rational agents maximize their (expected) utility:

- decision theory
- Markov decision processes (MDPs)
- reinforcement learning
- AI planning
- ...

Rational Agents in Game Theory

Situation in game theory:



Rational Agents in Game Theory

Multiple rational agents interacting in strategic decision situations.

- resulting utility depends on what other agents do.
- all agents know that other agents are rational (this is even common knowledge).

Interesting questions:

- how to model such strategic situations
- how to solve such strategic situations
- how to design games that have desired solutions

Game theory is the study and analysis of such strategic decision situations.

History of Game Theory

- originally part of mathematics and theoretical economics
- today ubiquitous
- here: artificial intelligence and computer science perspective
  - rationality assumptions ("homo economicus") more warranted for artificial agents than for humans
  - interesting algorithmic questions

2 Application Examples

Board and Card Games

Two-player board and card games:

- very special
- whatever is good for one player is bad for the other (strictly competitive games)
- recent visible success: Poker (no-limit, heads-up, hold'em)

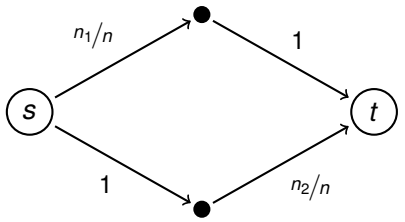
Auctions

Auctions: Think of eBay, Google AdWords, ...

- setting: one object should be allocated to one out of a number of bidders.
- questions:
  - what bidding protocol to use?
  - who is the winner?
  - what does the winning bidder have to pay?

Congestion Games

Congestion games: road network with travel costs dependent on the number of agents choosing a particular road

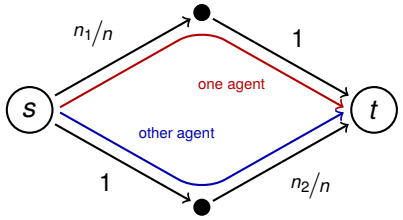


Question: Assume that there are $n = 2$ agents.
Which routes will they choose?

Average travel cost per agent: ?

Congestion Games

Congestion games: road network with travel costs dependent on the number of agents choosing a particular road

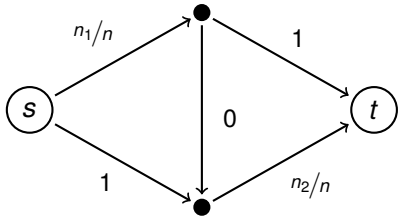


Question: Assume that there are $n = 2$ agents.
Which routes will they choose?

Average travel cost per agent: 1.5

Congestion Games

Congestion games: road network with travel costs dependent on the number of agents choosing a particular road

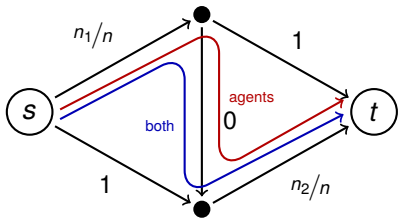


Question: Assume that there are $n = 2$ agents.
Which routes will they choose now (with free new road)?

Average travel cost per agent: ?

Congestion Games

Congestion games: road network with travel costs dependent on the number of agents choosing a particular road



Question: Assume that there are $n = 2$ agents.
Which routes will they choose now (with free new road)?

Average travel cost per agent: $2 > 1.5$

Security Games

Security games:

- setting: a facility (e. g., an airport) has to be guarded to avoid attacks
- possible methods:
  - visit all critical places
  - choose the places probabilistically
  - find a probability distribution for the routing that minimizes expected damage even under the assumption that the attacker can observe the guards

Elections

- setting: a set of alternatives (candidates) and a set of voters, determine winner or ranking
- questions:
  - what questions to ask?
  - how to determine a winner / ranking?
  - what is the computational complexity of determining a winner?
  - can the protocol be made manipulation-safe?

3 Rationality

Rationality

Rationality:

- General assumption: All players want to maximize their own utility and nothing else.
- Contrasts:
  - Altruistic agents want to maximize utility of other agents
  - Cooperative agents want to maximize group utility
  - Byzantine agents want to minimize utility of other agents

Limitations:

- agents may not foresee all consequences of their decisions (bounded rationality)
- agents may not know all relevant information about the game structure (incomplete information)
- agents may not know all relevant information about the current state of the game (imperfect information)

4 Course Outline

Course Outline

- strategic games
- extensive games
- repeated games
- imperfect information games and Poker
- social choice theory
- mechanism design

5 Let's Play a Game

Beauty Contest

We play a game called "Beauty Contest".

Rules

Everybody chooses a natural number n with $1 \leq n \leq 100$.
The players that come closest to $2/3$ of the average win.

Now it's your turn!