Introduction to Game Theory

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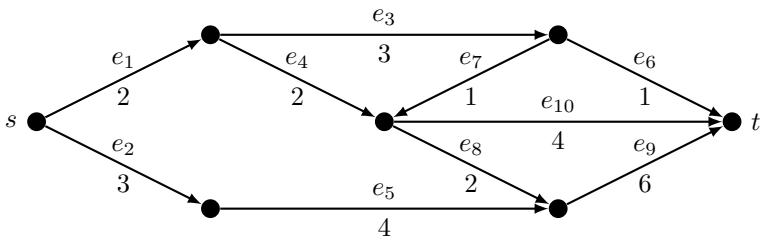
Exercise Sheet 9

Due: Thursday, June 4, 2019

Send your solution to `schultet@informatik.uni-freiburg.de` (PDF only) or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.

Exercise 9.1 (Network routing as VCG-mechanism, 2 points)

Let $G = (V, E)$ be a directed graph. Every edge $e \in E$ belongs to a player e and generates cost c_e when being used. We want to rent a path between the two nodes s and t. The set of alternatives A contains all paths between s and t. Player e has cost c_e, if edge e lies on the chosen path p, zero otherwise. Maximization of social welfare means minimizing $\sum_{e \in p} c_e$ over all paths p from s to t. Which alternatives does the VCG-mechanism choose in the following concrete example? Which payments result from this? Please justify your answers.



Exercise 9.2 (Top trading cycle method, 3 points)

(a) Apply the top trading cycle algorithm to the following problem and state what happens in the iterations:

- *Player 1:* $1 \lhd_1 4 \lhd_1 2 \lhd_1 3$
- *Player 2:* $3 \lhd_2 2 \lhd_2 1 \lhd_2 4$
- *Player 3:* $2 \lhd_3 3 \lhd_3 4 \lhd_3 1$
- *Player 4:* $2 \lhd_4 1 \lhd_4 4 \lhd_4 3$

Preferences are given from lowest (left) to highest (right).

Exercise 9.3 (Stable matchings, 3 points)

Apply the deferred acceptance algorithm with male proposals to the following problem and state what happens in the iterations:

- *Man 1:* $w_4 \prec_{m_1} w_3 \prec_{m_1} w_1 \prec_{m_1} w_2$
- *Man 2:* $w_3 \prec_{m_2} w_2 \prec_{m_2} w_1 \prec_{m_2} w_4$
- *Man 3:* $w_4 \prec_{m_3} w_2 \prec_{m_3} w_3 \prec_{m_3} w_1$
- *Man 4:* $w_4 \prec_{m_4} w_1 \prec_{m_4} w_3 \prec_{m_4} w_2$
- *Woman 1:* $m_4 \prec_{w_1} m_2 \prec_{w_1} m_3 \prec_{w_1} m_1$
- *Woman 2:* $m_2 \prec_{w_2} m_1 \prec_{w_2} m_4 \prec_{w_2} m_3$
- *Woman 3:* $m_1 \prec_{w_3} m_3 \prec_{w_3} m_2 \prec_{w_3} m_4$
- *Woman 4:* $m_4 \prec_{w_4} m_1 \prec_{w_4} m_2 \prec_{w_4} m_3$

Preferences are given from lowest (left) to highest (right).