Introduction to Game Theory

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Exercise Sheet 8

Due: Thursday, June 27, 2019

Send your solution to `schultet@informatik.uni-freiburg.de` (PDF only) or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.

Exercise 8.1 (Single peaked preferences, 1 + 2 points)

Allan (A), Mark (M), and Kenneth (K) discuss how much time to invest in collective preparations for their upcoming exam in game theory. Their valuations over the amount of time $x \in \mathbb{R}^{>0}$ (in hours) to invest are as follows:

$$v_A(x) = -\frac{7}{3} + \frac{7}{3}x - \frac{1}{15}x^2 \qquad v_M(x) = -\frac{1}{2}x + 20 \qquad v_K(x) = 4x - \frac{1}{5}x^2$$

To agree on a fixed amount of time $x \in [5, 30]$, Allan, Mark, and Kenneth take a vote in which each of them submits a single peaked preference relation.

(a) On what amount of time will they agree using the median rule?

(b) Show that the median rule is not incentive compatible when the preference relations are not restricted to be single peaked.

Exercise 8.2 (Vickrey-Clarke-Groves Mechanism; 2+3 points)

In a *k-item auction*, k identical items are to be sold. Each bidder $i = 1, \dots, n$ can get at most one of the items and has a privately known valuation w_i for the item. For simplicity, assume that $w_1 > w_2 > \dots > w_n$. The set of alternatives $A = N_k$ consists of all k-ary subsets of players. Each alternative represents the players who will receive an item.

(a) Formalize the k-item auction as a VCG mechanism $\mathcal{M} = \langle f, (p_i)_{i \in N} \rangle$ that uses *Clarke pivot functions*.

(b) Consider the mechanism $\mathcal{M}' = \langle f', (p'_i)_{i \in N} \rangle$ implementing a k-item auction, with

- social choice function $f'(v_1, \dots, v_n) = \{i \in N \mid 1 \leq i \leq k\}$, and
- payment functions $p'_i(a) = \begin{cases} w_{i+1}, & \text{if } i \in a, \\ 0, & \text{otherwise,} \end{cases}$ for all $a \in A$.

Here, the i-th highest bidding winner has to pay the $(i + 1)$-st highest bid, i. e., the highest bidding player pays the second highest bid, the second highest bidder pays the third highest bid, and so on. Non-winning players pay nothing. Construct a counterexample with only three bidders that proves that $\mathcal{M}'$ is *not* incentive compatible.