Introduction to Game Theory

B. Nebel, R. Mattmüller
T. Schulte, D. Speck
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University of Freiburg
Department of Computer Science

Exercise Sheet 7

Due: Thursday, June 20, 2019

Send your solution to `schultet@informatik.uni-freiburg.de` (PDF only) or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.

Exercise 7.1 (Schulze method, 2 points)
Consider the following preference relations:

20 voters have the preference $b \prec_i c \prec_i e \prec_i d \prec_i a$
10 voters have the preference $d \prec_i e \prec_i c \prec_i b \prec_i a$
15 voters have the preference $b \prec_i d \prec_i a \prec_i e \prec_i c$
12 voters have the preference $a \prec_i b \prec_i c \prec_i e \prec_i d$
13 voters have the preference $a \prec_i e \prec_i c \prec_i d \prec_i b$

Determine the set of possible winners according to the Schulze-method[1] for the preference relations of the voters given above.

Exercise 7.2 (Properties of voting procedures, 3 points)
Consider the voting procedures **plurality vote**, **instant runoff voting**, and the **Borda count**. Again, we assume that ties are broken in favor of the candidate with the lower index. Moreover, $|A| \geq 3$. Consider the following properties:
Majority criterion:

$$\text{If for more than half of the voters } i, b \prec_i a \text{ for all } b \in A \setminus \{a\}, \text{ then } f(\prec_1, \dots, \prec_n) = a.$$

Reversal symmetry:

$$\text{If } f(\prec_1, \dots, \prec_n) = a \text{ and } a \prec'_i b \text{ iff } b \prec_i a \text{ for all } i = 1, \dots, n \text{ and } a, b \in A, \text{ then } f(\prec'_1, \dots, \prec'_n) \neq a.$$

Incentive compatibility:

$$f(\prec_1, \dots, \prec'_i, \dots, \prec_n) \preceq_i f(\prec_1, \dots, \prec_i, \dots, \prec_n) \text{ for all } \prec_1, \dots, \prec_n, \prec'_i \in L.$$

For each of the nine combinations of voting procedure f and property P, show that f satisfies P or give a counterexample.

Exercise 7.3 (May's theorem, 1 + 1 + 1 points)
May's theorem: A social choice function $f : L^n \to A$ for a set of two alternatives $A = \{x, y\}$ satisfies anonymity, neutrality and monotonicity iff it is the plurality method (i.e., $f(\prec_1, \dots, \prec_n) = x$ iff $\#\{i \mid y \prec_i x\} \geq \frac{n}{2}$). We assume n is odd to avoid tie-breaking issues that could violate neutrality. Show that each of the three conditions is necessary for May's theorem: For each condition, find a counterexample (a social choice function) that fulfills all other conditions but the one in question and that is not the plurality method.

(a) anonymity, i.e., $f(\prec_1, \dots, \prec_n) = f(\prec_{\pi(1)}, \dots, \prec_{\pi(n)})$ for all permutations π of the voters $\{1, \dots, n\}$.

(b) neutrality, i.e., $f(\prec_1, \dots, \prec_n) = x$ iff $f(\prec'_1, \dots, \prec'_n) = y$, where $x \prec'_i y$ iff $y \prec_i x$ for all $i = 1, \dots, n$.

(c) monotonicity, i.e., if $f(\prec_1, \dots, \prec_n) = x$, then also $f(\prec'_1, \dots, \prec'_n) = x$, where $\prec'_i = \prec_i$ for $i \neq I$ for some voter I such that $x \prec_I y$ and $y \prec'_I x$.

[1] `http://en.wikipedia.org/wiki/Schulze_method`