Introduction to Game Theory

B. Nebel, R. Mattmüller
T. Schulte, D. Speck
Summer semester 2019

University of Freiburg
Department of Computer Science

Exercise Sheet 6

Due: Thursday, June 6, 2019

Send your solution to `schultet@informatik.uni-freiburg.de` (PDF only) or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.

Exercise 6.1 (Voting procedures, 4 points)

For the following preference relations, determine the winners according to the **plurality vote**, **instant runoff voting**, **Borda count**, and **Coombs method**[1] (for simplicity, we assume that ties are broken in favor of the candidate with the lower index):

2 voters have the preference:	$a_2 \succ a_4 \succ a_3 \succ a_5 \succ a_1$
3 voters have the preference:	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$
1 voter has the preference:	$a_4 \succ a_2 \succ a_5 \succ a_1 \succ a_3$
2 voters have the preference:	$a_5 \succ a_3 \succ a_4 \succ a_2 \succ a_1$

Exercise 6.2 (Social welfare functions: unanimity, 2 + 2 points)

A social welfare function $F : L^n \to L$ satisfies

- **total unanimity** if for all $\prec \in L, F(\prec, \dots, \prec) = \prec$.
- **partial unanimity** if for all $\prec_1, \prec_2, \dots, \prec_n \in L, a, b \in A$,

$$a \prec_i b \text{ for all } i = 1, \dots, n \implies a \prec b, \text{ where } \prec := F(\prec_1, \dots, \prec_n).$$

(a) Proof that *partial unanimity* implies *total unanimity*.

(b) Proof by counter-example that *total unanimity* does not imply *partial unanimity*.
Hint: specify a social welfare function F that satisfies *total unanimity* but does not satisfy *partial unanimity*.

[1] https://en.wikipedia.org/wiki/Coombs%27_method