## Introduction to Game Theory

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## Exercise Sheet 4

## Due: Thursday, May 23, 2019

Send your solution to schultet@informatik.uni-freiburg.de or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.

Exercise 4.1 (Linear Complementarity Problem, $1+1$ points)
Consider the strategic game given by the following payoff matrix:
Player 2

Player $1 b$

|  | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | :---: |
| $a$ | 0,0 | 3,1 | 3,3 |
| $b$ | 1,1 | 0,0 | 1,3 |
| $c$ | 1,1 | 1,1 | 0,0 |
|  |  |  |  |

(a) For the following pair of support sets formulate the corresponding linear program: $(\operatorname{supp}(\alpha), \operatorname{supp}(\beta))=(\{a, b, c\},\{x, y, z\})$.
(b) Solve the linear program and provide values for each $\alpha\left(a_{1}\right)$ and $\beta\left(a_{2}\right), a_{1} \in\{a, b, c\}, a_{2} \in$ $\{x, y, z\}$. What is the expected payoff $(u, v)$ of the NE computed above?

Exercise 4.2 (Correlated equilibria, 3 points)
Consider the three player game with the following payoff matrix (Player 1 chooses one of the two rows, player 2 chooses one of the two columns, and player 3 chooses one of the three tables.)


A


B

|  | $L$ | $R$ |
| :---: | :---: | :---: |
| $T$ | $0,0,0$ | $0,0,0$ |
| $B$ | $0,1,0$ | $0,0,3$ |
|  |  |  |

C
(a) Show that the pure strategy equilibrium payoffs are $(1,0,0),(0,1,0)$, and $(0,0,0)$.
(b) Show that there is a correlated equilibrium in which player 3 chooses $B$ and players 1 and 2 play $(T, L)$ and $(B, R)$ with equal probabilities.
(c) Explain the sense in which player 3 prefers not to have the information that players 1 and 2 use to coordinate their actions.

Exercise 4.3 (Induced Strategic Game, $1+2$ points)
Consider the two player extensive form game defined by the following game tree.

(a) Specify the induced strategic game.
(b) Determine all Nash equilibria and decide for each one whether it is subgame perfect or not.

