Introduction to Game Theory

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Summer semester 2019

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Exercise Sheet 3

Due: Thursday, May 16th, 2019

Send your solution to `schultet@informatik.uni-freiburg.de` or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.

Exercise 3.1 (Nash equilibria in zero-sum games, 1 point)

Prove the following claim or give a counterexample: If G is a two-player zero-sum game that has a Nash equilibrium with payoff v for player 1 then every strategy profile in G with payoff v for player 1 is a Nash equilibrium.

Exercise 3.2 (Kakutani's fixed point theorem, 1+1+1+1 points)

Let X be a compact, convex, non-empty subset of $\mathbb{R}^n$ and let $f : X \to 2^X$ be a set-valued function for which

- for each $x \in X$, the set $f(x)$ is nonempty and convex;
- the graph of f is closed (i.e. for all sequences $\{x_k\}$ and $\{y_k\}$ such that $y_k \in f(x_k)$ for all k, $x_k \to x$, and $y_k \to y$, we have $y \in f(x)$).

Then there exists $x^* \in X$ such that $x^* \in f(x^*)$.

Show that each of the following four conditions is necessary for Kakutani's theorem by constructing a counter-example with $n = 1$ in each of the four cases.

(a) X is compact.

(b) X is convex.

(c) $f(x)$ is convex for each $x \in X$.

(d) f has a closed graph.

Exercise 3.3 (Linear Programming, 2 + 1 points)

Rock-paper-scissors can be formalized as a two player zero-sum game as follows:

		Player 2		
		rock	*paper*	*scissors*
Player 1	*rock*	$0, 0$	$-1, 1$	$1, -1$
	paper	$1, -1$	$0, 0$	$-1, 1$
	scissors	$-1, 1$	$1, -1$	$0, 0$

(a) Formulate the corresponding linear program.

(b) Use a linear programming solver (e.g. lp_solve[1]) to compute a solution, or guess a solution and verify it manually.

[1] `http://lpsolve.sourceforge.net/5.5/`