

# Introduction to Game Theory

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## Exercise Sheet 1

**Due: Thursday, May 2nd, 2019**

Send your solution to `schultet@informatik.uni-freiburg.de` or submit a hardcopy before the lecture. If you submit by email, please start the subject line of your message with `[gt19]`.

### Exercise 1.1 (Teamwork, 4 additional points)

We believe that working in groups is a strictly dominant strategy for you. If you submit your solution as a group of three, you will automatically receive four additional points. Otherwise, you can collect these points by sending an email to `schultet@informatik.uni-freiburg.de` with your name so that we can form groups of students. Please use as subject of the email: `[gt19] Teamwork`.

### Exercise 1.2 (Street network, 2+1 points)

Consider the following, slightly modified instance (Figure 1(a)) of the routing example (Figure 1(b)) that was introduced in the lecture.

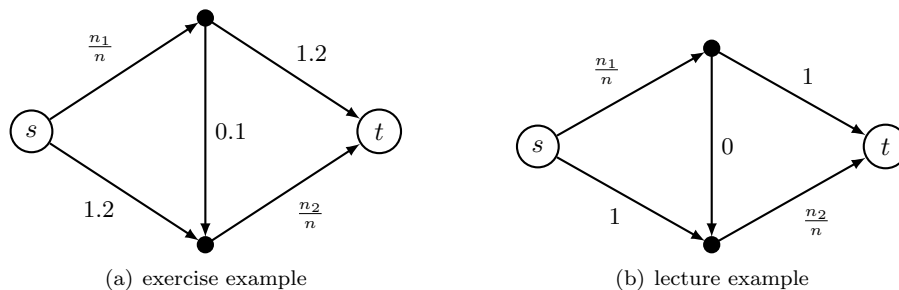


Figure 1: Streets of a street network

- Formalize the special case of two agents as a strategic game and identify all strictly or weakly dominated actions as well as all Nash equilibria.
- What changes occur in both games, when more than two agents take part?

**Exercise 1.3** (Game of thrones, 1+1 points)

On the far away continent “Westeros”, the Night King and his White Walkers threaten to invade the realm of the humans from the north.

Both Queen Cersei and Queen Daenerys want to keep or conquer the Iron Throne, respectively. However, both are also interested in protecting the living from the White Walkers. Depending on their strategies, they will either fight independently, or they might join forces. In any case, it could be considered wise to decide beforehand who of them will be the new queen after the White Walkers are defeated.

- If both **claim** to be queen, they will start fighting against each other and the Night King will prevail.
- If both of them **yield**, however, someone else, for instance the smart Tyrion, will fill the power vacuum and take the Iron Throne.
- If only one of them steps back to let the other have the crown, the Night King will quite certainly be defeated.

To make a long story short, this situation can be considered a strategic game with the following payoff matrix:

		Daenerys	
		yield	claim
Cersei	yield	3, 3	1, 5
	claim	5, 1	0, 0

Let us assume next that the two self-proclaimed queens have decided to join their forces—in the literal sense, that is, by putting their men together. Still, they can decide where the battle field between their joint forces and the White Walkers will be. They can let the White Walkers pass through the North without a fight to a battle field further south, near King’s Landing, or to seek the epic decisive battle in the North at Winterfell. Cersei prefers fighting in the North, whereas Daenerys prefers fighting in the South.

The following payoff matrix captures this problematic situation:

		Daenerys	
		North	South
Cersei	North	4, 2	0, 0
	South	0, 0	2, 4

- How many Nash-equilibria does the claim-yield game have and, if any, which ones?
- How many Nash-equilibria does the North-South game have and, if any, which ones?

**Exercise 1.4** (Best response function, 3 points)

Let  $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$  with  $N = \{1, 2\}$ ,  $A_1 = A_2 = \mathbb{R}^{\geq 0}$ ,  $u_1(a_1, a_2) = a_1(a_2 - a_1)$  and  $u_2(a_1, a_2) = a_2(1 - \frac{1}{2}a_1 - a_2)$  for all  $(a_1, a_2) \in A$ .

Define all Nash equilibria of this game by constructing and analyzing the best response function of both players.

The exercise sheets may and should be worked on and handed in in groups of two to three students. Please indicate all names on your solution.