Dynamic Epistemic Logic

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Exercise Sheet 2
Due: May 9th, 2019, 16:00

Exercise 2.1 (Sally-Anne Test, 1+2)

We want to model the Sally-Anne test (you can find its description on page 4 of the lecture slides 1), which involves belief instead of knowledge. We use the language $\mathscr{L}_K$ to talk about belief by simply reinterpreting the meaning of $K_i\phi$ as *agent i believes ϕ*. However, we evaluate these formulas on *doxastic* KD45 models, i.e., Kripke models where the accessibility relations are serial, transitive and Euclidean. As usual, $K_i\phi$ is true in some given world w iff ϕ is true in all the worlds w' that are accessible from world w for agent i. Since we don't require accessibility relations to be reflexive, agents can have false beliefs.

(a) Model the situation in which the marble is in Anne's Box (and Anne knows this) but where Sally still believes that it is in the basket as a KD45 model.

(b) According to your model, does Anne believe that Sally believes that the marble is in the basket? Does Sally believe that Anne believes that the marble is in the basket? Answer these questions by checking whether the corresponding formulas are satisfied in your model.

Exercise 2.2 (Satisfiability in S5, 1+1+1)

Show that the following $\mathcal{L}_K$ formulas are satisfiable in S5.

(a) $K_a(p \wedge \hat{K}_b \neg p)$

(b) $K_a K_b p \wedge \neg K_b K_a p$

(c) $\neg p \wedge \hat{K}_2 K_1 p \wedge \hat{K}_1 K_2 p \wedge K_2 \hat{K}_1 p \wedge K_1 \hat{K}_2 p$

Exercise 2.3 (Validity of S5 axioms, 2+2+2)

Show that the following $\mathcal{L}_K$ formulas are valid in S5.

(a) $K_a\phi \rightarrow \phi$

(b) $K_a\phi \rightarrow K_a K_a \phi$

(c) $\neg K_a\phi \rightarrow K_a \neg K_a \phi$