## Multi-Agent Systems

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# Course outline

- 1 Introduction
- 2 Agent-Based Simulation
- 3 Agent Architectures
- 4 Beliefs, Desires, Intentions
- 5 Norms and Duties
- 6 Communication and Argumentation
- 7 Coordination and Decision Making
  - Distributed Constraint Satisfaction
  - Auctions and Markets
  - Cooperative Game Theory



#### Last time

Distributed Constraint Satisfaction as a means to implement distributed problem solving: Each agent tries to assign some value to its private variable under consideration of its known constraints and on the feedback it receives from the other agents.

#### Today

Agents display selfishness, i.e., they have preferences (of probably different strengths) and compete with each other about rare goods. We will employ the metaphors of Auctions and Markets.



- A seller is auctioning one item to a set of bidders.
- Each bidder has an intrinsic value (or true value) for the item.



- Bidders have independent, private values, i.e., agents are uncertain about the other agents' intrinsic values.
- Sellers do not have a good estimate of the bidders' true value of an item.



- Sellers want to maximize the price at which the good is allocated: "Which type of auction should I set up for my good?"
- Bidders want to minimize the price at which the good is allocated: "Which price should I bid?"

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Auctions: Types

#### Examples

- Ascending-bid auctions (English auction)
- Descending-bid auction (Dutch auction)
- First-price sealed-bid auctions
- Second-price sealed-bid auctions (Vickrey auctions)
- and there are many more.

#### Dimensions

- Bidding rules: How are offers made?
- Clearing rules: When do which trades occur as a function of the bidding?
- Information rules: Who knows what when about the state of the auction?





- Bidding rule: Auctioneer starts off by an initial price for the good. Agents are invited to bid more.
- Clearing rule: No agent is willing to raise current bid, and the highest bidder wins.
- Information rule: Every agent can see what the others are bidding.



- Bidding rule: Auctioneer starts off by an initial price for the good. Auctioneer continually lowers the offer price.
- Clearing rule: An agent makes a bid for the good which is equal to the current offer price.
- Information rule: Every agent can see what the others are bidding.



- Bidding rule: Bidders submit to the auctioneer a bid for the good.
- Clearing rule: Winner is the agent who made highest bid. Winner pays the price (s)he bid.
- Information rule: No agent can see what the others are bidding.



- Bidding rule: Bidders submit to the auctioneer a bid for the good.
- Clearing rule: Winner is the agent who made highest bid. Winner pays the second-highest price bid.
- Information rule: No agent can see what the others are bidding.



- Which auction protocol would you prefer as a bidder?
- Which auction protocol would you prefer as a auctioneer?

- Descending-bid auctions correspond to First-price sealed-bid auctions.
- Ascending-bid auctions correspond to Second-price sealed-bid auctions.
- In Second-price sealed-bid auctions bidding one's true value is a dominant strategy.
- 4 In First-price sealed-bid auctions  $s(v_i) = (\frac{n-1}{n})v_i$  is an equilibrium strategy.
- The money the seller can expect from a First-price sealed-bid auction is the same it can expect from a Second-price sealed-bid auction.



#### Proposition

Descending-bid auctions correspond to First-price sealed-bid auctions.

#### Proof

- Consider a descending-bid auction:
- Seller is lowering the price from its high initial starting point.
- No bidder says anything until finally s.o. accepts the bid and pays the current price.
- For bidder *i*, there's a first price  $b_i$  (s)he accepts.
- The price b<sub>i</sub> plays the role of bidder i's bid in a first-price sealed-bid auction.



#### Proposition

Ascending-bid auctions correspond to Second-price sealed-bid auctions.

#### Proof

- Consider an ascending-bid auction.
- The seller steadily raises the price and bidders gradually drop out.
- The winner of the auction is the last bidder remaining, and (s)he pays the price at which the second-to-last bidder dropped out.

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# Second-price sealed-bid auctions as games: Basics

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- Set of players  $\mathcal{N} = \{1, 2, \dots, n\}$  (the bidders)
- The true value *v<sub>i</sub>* of player *i* for the item
- Strategy  $s(v_i) = b_i$  is the bid of player *i* as a function of  $v_i$
- Payoff: If  $b_i$  is not the winning bid, then the payoff to *i* is 0. If  $b_i$  is the winning bid, and some other  $b_j$  is the second-place bid, then the payoff to *i* is  $g(v_i) = v_i b_j$ .

# Dominant strategy for second-price sealed-bid auctions



#### Proposition

In a second-price sealed-bid auction, it is a dominant strategy (i.e., best strategy regardless of what strategy everyone else is using) for each bidder *i* to choose a bid  $b_i = v_i$ .



#### Proof

- Case *i*'s valuation v<sub>i</sub> is larger than the highest of the other bidders' bids.
  - If *i* bids  $v_i$  it wins and pays the next-highest bid amount.
  - If *i* bids more than  $v_i$ , *i* would still win and pay the same.
  - If i bids less than v<sub>i</sub>, it either still wins and pays the same, or it looses and receives zero payoff.
- Case i's valuation v<sub>i</sub> smaller than the highest of the other bidders' bids.
  - If *i* bids  $v_i$ , it looses and gains zero payoff.
  - If i bids more than v, i either looses and gains zero payoff or wins and gains zero (or less) payoff.
  - If *i* bid less than *v*, it still looses and gains zero payoff.

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#### Assumptions

- Common knowledge between two bidders: v<sub>1</sub>, v<sub>2</sub> are independently and uniformly distributed between 0 and 1.
- Both bidders use the same strategy  $s(\cdot)$
- $s(\cdot)$  is strictly increasing and differential (so, if two bidders have different values, then they will bid differently).

• 
$$s(v) \leq v$$
 for all  $v, s(0) = 0$ 

#### Sample Strategies

- s(v) = v: bid your true valuation
- s(v) = cv, c < 1: scale down your bid a bit

...

#### Observations

- If bidder *i* has a true value of v<sub>i</sub> (drawn from [0, 1]), the probability that *i* wins is v<sub>i</sub>
- If bidder *i* wins, it receives the payoff  $v_i s(v_i)$
- Thus, the expected payoff for bidder *i* is

$$g(v_i) = v_i(v_i - s(v_i))$$

Note that  $s(v_i) = v_i$  is not a dominant strategy in first-price auctions: If *i* loses, it gets 0 payoff, and if *i* wins, the payoff is  $(v_i - v_i) = 0$ , too.





#### Equilibrium strategy

- For s(·) to be an equilibrium strategy means that for each bidder *i*, there is no incentive to deviate from strategy s(·) if its competitor is also using strategy s(·).
- The incentive to deviate can be simulated by pretending other deviations v from v<sub>i</sub>.
- Formally: The expected payoff  $g(v) = v(v_i s(v))$  should be maximal for  $v = v_i$ .

# Equilibrium in First-Price sealed-bid Auctions: Two Bidders



#### Equilibrium strategy

# Equilibrium in First-Price sealed-bid Auctions: n Bidders

#### Equilibrium strategy

Expected payoff for bidder i

$$G(v_i) = v_i^{n-1}(v_i - s(v_i))$$

Setting  $G'(v_i) = 0$ , we have

$$(n-1)v^{n-2}v_i - (n-1)v^{n-2}s(v_i) - v_i^{n-1}s'(v_i) = 0$$

- Then we obtain  $s'(v_i) = (n-1)(1 \frac{s(v_i)}{v_i})$
- This differential equation is solved by

$$s(v_i) = \left(\frac{n-1}{n}\right)v_i$$



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#### The seller's perspective

Which auction type should the seller choose to get as much money as possible?

#### From what we know so far

- Two competing forces
  - Second-price sealed-bid auction: Seller charges second-highest bid
  - First-price sealed-bid auction: Bidders reduce their bids

#### Proposition

Suppose *n* numbers are drawn independently from the uniform distribution on the interval [0, 1] and then sorted from smallest to largest. The expected value of the number in the *k*th position on this list is  $\frac{k}{n+1}$ .

#### Application to Second-price Auctions

The seller gets the second-highest value, i.e., at position n-1. Thus the expected value is  $\frac{n-1}{n+1}$ .

#### Application to First-price Auctions

The winning bidder bids  $\frac{(n-1)}{n}v_i$  (Equilibrium strategy). Using the propositon,  $v_i = \frac{n}{n+1}$ . Hence  $\frac{(n-1)}{n}\frac{n}{n+1} = \frac{n-1}{n+1}$ 



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- Application to ressource assignment: Is it wise to just sequentially run auctions to assign several items to several agents?



Matching markets generalize this setting to many items and many bidders.



- Origin: Economics, operation research
- Agents have different preferences for different kinds of goods
- Prices determine the allocation of goods ...
- ...and socially optimal allocations emerge.





- Bipartite graphs, paths, and matchings
- 2 Market Framework
- 3 Algorithm for computing market-clearing prices



#### Definition

A graph G = (V, E) is a bipartite graph iff there are two disjoint sets  $A, B \subseteq V, A \cup B = V$  such that every edge  $e \in E$  connects a vertex in A to one in B. G is balanced iff |A| = |B|.

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#### Matching

Given a graph G = (V, E), a matching  $M \subseteq E$  in *G* is a set of edges such that no two edges share a common vertex.

#### Maximum matching

A matching is a maximum matching iff it contains the largest possible number of edges.

#### Perfect matching

A perfect matching in a balanced bipartite graph  $G = (A \cup B, E)$  is a set of edges between nodes in *A* and nodes in *B* such that each node is endpoint of exactly one edge.



#### Free Vertex

A vertex  $v \in V$  is a free vertex w.r.t. to a matching *M* iff no edge from *M* is incident to *v*.



#### Path

A path  $P \subseteq E$  in a graph G = (V, E) is a sequence of edges which connect a sequence of distinct vertices.

#### Alternating Path

P is an alternating path w.r.t. M, iff P is a path in G and for every two subsequent edges on P it holds that one is in M and the other is not.

#### Augmenting Path

An alternating path *P* is an augmenting path iff its start and end vertices are free.



function Maximum-Matching(G)

 $M = \emptyset$ 

#### repeat

P = Augmenting-Path(M, G)

 $M = M \otimes P$  [Symmetric difference,  $A \otimes B = \{x | x \in A \oplus x \in B\}$ ]

until 
$$P = \emptyset$$

#### end function

function Augmenting-Path(M, G)

Direct unmatched edges  $A \rightarrow B$ , matched  $B \rightarrow A$ 

Add vertices s, t, connect s to free vertices in A and the

#### free vertices of B to t

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Run BFS to find a shortest path P from s to t
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return  $P \setminus \{s, t\}$ 

#### end function

# Example: Compute maximum matching





# Example: Compute maximum matching II









# Example: Compute maximum matching IV





If the graph has a perfect matching, a component can solve the allocation problem using the maximum-matching algorithm in a centralized way.





- 1 Bipartite graphs, paths, and matchings
- 2 Market Framework
- 3 Algorithm for computing market-clearing prices



- Set S of Sellers (each with an object to sell)
- Set B of Buyers (each with valuation for each object)
- *v<sub>ij</sub>*: Valuation of buyer *j* for the object held by seller *i*
- *p<sub>i</sub>*: Price fixed by seller *i*
- $v_{ij} p_i$ : Buyer j's payoff if he/she/it buys from seller i
- Note: If v<sub>ij</sub> − p<sub>i</sub> is negative for some j and all i then j does no transaction and receives payoff 0.



#### **Definition: Preferred sellers**

The set of sellers that maximize *j*'s payoff is called preferred sellers of buyer *j* (provided the payoffs are non-negative).

#### Definition: Preferred seller graph

The preferred seller graph (PSG) is the bipartite graph  $G = (S \cup B, E)$  s.th.  $(i, j) \in E$  iff *i* is a preferred seller of *j*.

#### Definition: Market-clearing prices

A set of prices is called market clearing iff the resulting preferred-seller graph has a perfect matching.



#### Definition: Neighbor Set

Given a balanced bipartite graph  $G = (S \cup B, E)$ . For any sets of nodes  $X \subseteq B$  (resp. S) a node  $v \in S$  (resp. B) is a neighbor of X iff it has an edge to some node in B.

The neighbor set of  $\mathcal{B}$ ,  $N(\mathcal{B})$ , is the set of all neighbors of  $\mathcal{B}$ .

#### Definition: Constricted Set

A set  $\mathcal{B}$  is a constricted set iff  $|\mathcal{B}| > |\mathcal{N}(\mathcal{B})|$ .





- Does the graph have a constricted set?
- Is there a perfect matching?
- Are the prices market clearing?



- **PSG** has perfect matching  $\Rightarrow$  prices clear the market.
- Some cooperation needed among sellers (or use the previously introduced centralized algorithm)
- Do the agents as a whole get what they want most?



- Higher prices do not improve the allocation.
- Payoff: the sellers' gain = the buyers' loss.



A preferred seller graph has a perfect matching iff it contains no constricted set.

Given a set of market-clearing prices. A perfect matching in the resulting preferred-seller graph has the maximum total valuation.

#### Proof

- Consider a set of market-clearing prices MCP, and let M be a perfect matching in the PSG
- Total payoff of buyers = Total Valuation Sum of all prices
- Since each buyer individually maximizes her payoff (PSG!), the total payoff of buyers is maximal, i.e., no other matching yields more payoff than *M* under the given *MCP*.
- The sum of prices is the same for all matchings Matchings that maximize total payoff also maximizes total valuation.



Given a set of market-clearing prices. A perfect matching in the resulting preferred-seller graph produces the maximum possible sum of payoffs to all sellers and buyers.

Very similar argument.





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#### Theorem

For any set of buyer valuations, there exists a set of market-clearing prices.

Let's construct a algorithm for searching for a set of market-clearing prices.

#### Algorithm (see [2])

- All sellers initially set prices to 0
- 2 Construct preferred-seller graph PSG
- 3 If PSG has a perfect matching, it's done
- 4 Else there must be a constricted set of buyers  ${\cal B}$
- 5 The neighbors N(B) raise their prices by one unit
- 6 Scale prices s.th. the minimum price is 0
- 7 Goto 2
- This algorithm terminates ⇔ it finds a set of prices that yield a PSG with a perfect matching ⇔ (by definition) it finds a set of market-clearing prices.
- Will it terminate?

# Example execution I





Note: In this case there are two constricted sets  $\{1,3\}$  and  $\{1,2,3\}$ . Either of them can be chosen.

## Example execution II





# Example execution III





# Example execution IV





Perfect matching: (1,a), (2,c), (3,b)



#### Definition: Potentials of buyers, sellers, and of the auction The potential of a buyer is the maximum payoff she can currently get from any seller. The potential of a seller is the current price she is charging. The potential energy of the auction is the sum of the potential of all buyers and sellers.

Note: The minimum payoff of a buyer (its potential) is 0. The minimum price a seller can charge (its potential) is 0. Thus, the potential energy of the auction must be greater or equal to 0.

The procedure always terminates.

#### Proof

- Initially, all sellers have potential 0 and buyers have potential according to their highest valuation. Thus, at the start  $P_0 \ge 0$ .
- In every round, when sellers in *N*(*B*) (*B* constricted set) raise their prices, each of these sellers' potential increases and the potential of buyers in *B* decreases.
- Since  $|\mathcal{B}| > |\mathcal{N}(\mathcal{B})|$ , the overall potential decreases.
- Because the potential cannot drop below 0, the procedure will terminate.



- Item a is the single item, the other items are dummies (to have a balanced bipartite graph)
- Item *a* increases its price until only one is left.
- Finally, 1 gets assigned to *a* and he/she/it must pay 2.
- Proposition: Market-Clearing Prices auction implements English auction (second-price sealed-bid auction).

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### Literature I



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- D. Easley, J. Kleinberg, Networks, Crowds, and Markets: Reasoning about a Highly Connected World, Cambridge University Press, 2010.