Multi-Agent Systems

Albert-Ludwigs-Universität Freiburg

Bernhard Nebel, Felix Lindner, and Thorsten Engesser

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Course outline

1. Introduction
2. Agent-Based Simulation
3. Agent Architectures
4. Beliefs, Desires, Intentions
   - The GOAL Agent Programming Language
   - Introduction to Modal Logics
     - Part I: Graphical Models, Kripke Models
     - Part II: Syntax & Semantics
     - Part III: Tableaux via Graph Rewriting
   - Epistemic Logic
   - BDI Logic
5. Norms and Duties
6. Communication and Argumentation
7. Coordination and Decision Making

Overview

- Recap
  - Situations from various domains (Programs, Knowledge, Belief, Desire, Obligation) can nicely be modeled using graphical models.
  - Kripke models formalize graphical models.
  - By constraining the accessibility relations of Kripke frames we obtain classes that correspond to above concepts (Knowledge, Belief etc.)
- Today
  - Introducing formal languages to talk about Kripke models and thus generally about Knowledge, Belief, Desire, Obligation ...

Modal Logics

Kripke Models

Kripke Frame

Given a countable set of edge labels $\mathcal{I}$, a Kripke Frame is a tuple (W, R) such that:

- W is a non-empty set of possible worlds, and
- $R : I \to 2^{W \times W}$ maps each $I \in \mathcal{I}$ to a binary relation $R(I)$ on W (called the accessibility relation of I).

Kripke Model

$M = (W, R, V)$ is a Kripke Model where:

- (W, R) is a Kripke frame, and
- $V : \mathcal{P} \to 2^W$ is called the valuation of a set of node labels $\mathcal{P}$.

Formulas of Modal Logic

The elements of our language are called formulas $\mathcal{F}$. These formulas talk about what is true at a given possible world w in a Kripke model $M = (W, R : \mathcal{I} \to 2^{W \times W}, V)$.

- Remember that $\mathcal{P}$ are the node labels in Kripke models. They constitute the atomic formulae in our language, e.g., $light\_on, sun\_shining$. Thus $\mathcal{P} \subseteq \mathcal{F}$.
- $\top, \bot \in \mathcal{F}$.
- If $\varphi \in \mathcal{F}$, then $\neg\varphi \in \mathcal{F}$.
- If $\varphi, \psi \in \mathcal{F}$ then $(\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \to \psi), (\varphi \leftrightarrow \psi) \in \mathcal{F}$
- If $\varphi \in \mathcal{F}$ and $I \in \mathcal{I}$, then $\mathbf{[I]}\varphi, \mathbf{\langle I\rangle}\varphi \in \mathcal{F}$.

Different Variants of Languages

- Alethic logic (Necessity): $\Box, \Diamond$
- Epistemic logic (Knowledge): $\mathbf{K}$, $\hat{\mathbf{K}}$
- Doxastic logic (Belief): $\mathbf{B}$, $\hat{\mathbf{B}}$
- Deontic logic (Obligation): $\mathbf{O}$, $\mathbf{P}$
- Multi-Agent Epistemic logic: Agent name as subscript, e.g., $\mathbf{K}_{mary}\hat{\mathbf{K}}_{john}sun\_shining$

Truth Conditions

Given a Kripke model M, a possible world w of M, and a formula φ. We define when φ is true at w, written $M, w \models \varphi$:

- $M, w \models p$ iff. $w \in V(p)$, for atomic formulae $p \in \mathcal{P}$.
- $M, w \not\models \bot$.
- $M, w \models \top$.
- $M, w \models \neg\varphi$ iff. $M, w \not\models \varphi$.
- $M, w \models (\varphi \wedge \psi)$ iff. $M, w \models \varphi$ and $M, w \models \psi$.
- $M, w \models (\varphi \vee \psi)$ iff. $M, w \models \varphi$ or $M, w \models \psi$.
- $M, w \models (\varphi \to \psi)$ iff. $M, w \not\models \varphi$ or $M, w \models \psi$.
- $M, w \models (\varphi \leftrightarrow \psi)$ iff. $M, w \models (\varphi \to \psi)$ and $M, w \models (\psi \to \varphi)$.
- $M, w \models \mathbf{[I]}\varphi$ iff. for every u: if $(w, u) \in R(I)$ then $M, u \models \varphi$.
- $M, w \models \mathbf{\langle I\rangle}\varphi$ iff. for some u: $(w, u) \in R(I)$ and $M, u \models \varphi$.

Duality

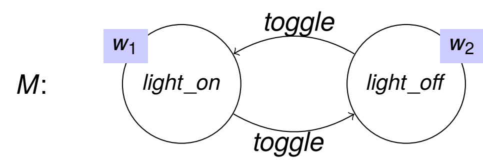
1. $M, w \models \mathbf{[I]}\varphi$ iff. $M, w \models \neg\mathbf{\langle I\rangle}\neg\varphi$
2. $M, w \models \mathbf{\langle I\rangle}\varphi$ iff. $M, w \models \neg\mathbf{[I]}\neg\varphi$

- To see that (1):
  - $M, w \models \mathbf{[I]}\varphi$
  - iff. for every u: if $(w, u) \in R(I)$ then $M, u \models \varphi$
  - iff. it is not the case that for some u: $(w, u) \in R(I)$ and $M, u \models \neg\varphi$
  - iff. not $M, w \models \mathbf{\langle I\rangle}\neg\varphi$
  - iff. $M, w \models \neg\mathbf{\langle I\rangle}\neg\varphi$

Model Checking

- Question: Is a given formula φ true in world w in model M?
- Input: A Kripke model M, a world w in M, and a formula φ.
- Output: "Yes" if $M, w \models \varphi$, "No" else.

Model Checking: Example



- $M, w_1 \models < toggle > \top \wedge [toggle][toggle]light\_on$

1 $M, w_1 \models < toggle > \top$

1.1 for some u: $(w_1, u) \in R(toggle)$ and $M, u \models \top$.

1.1.1 we find $(w_1, w_2) \in R(toggle)$ and $M, w_2 \models \top$.☺

2 $M, w_1 \models [toggle][toggle]light\_on$

2.1 for every u: if $(w_1, u) \in R(toggle)$ then $M, u \models [toggle]light\_on$

2.1.1 $M, w_2 \models [toggle]light\_on$.

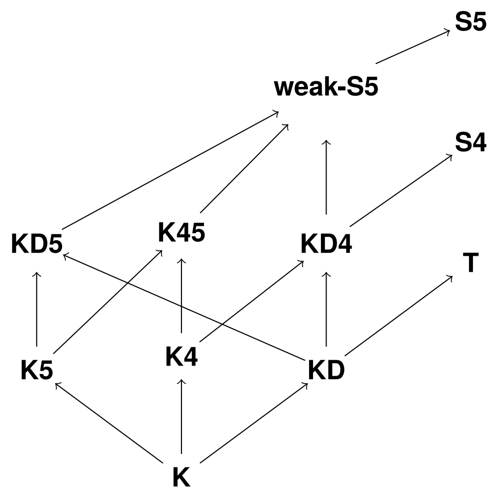
2.1.1.1 for every u: if $(w_2, u) \in R(toggle)$ then $M, u \models light\_on$

2.1.1.1.1 $M, w_1 \models light\_on$. ☺

Validity in a Class of Frames

- We say that a formula φ is valid in a class of frames **C** (one of **K**, **T**, **D**, **4**, **5**, and combinations thereof), written $\models_{\mathbf{C}} \varphi$, iff. $(W, R, V), w \models \varphi$
  - for every frame (W, R),
  - every valuation V over (W, R),
  - every world w in W.
- A formula φ entails ψ in the class **C** (written $\varphi \models_{\mathbf{C}} \psi$) iff. for every model M in **C** and every possible world w of M:
  - if $M, w \models_{\mathbf{C}} \varphi$ then $M, w \models_{\mathbf{C}} \psi$
- Entailment can be reduced to validity:
  - $\varphi \models_{\mathbf{C}} \psi$ iff. $\models_{\mathbf{C}} \varphi \rightarrow \psi$

A Lattice of Classes



Validity in a Class of Frames

- Valid formulas give us an idea of how the classes differ, and thus what is and is not specific to the general behavior of our modalities (Knowledge, Belief, Obligation etc.).
- Correspondences between classes of frames and formulas
  - **[I]**$(\varphi \to \psi) \to ($**[I]**$\varphi \to$ **[I]**$\psi)$ (for every formulae φ, ψ) is **K**-valid (valid in the class of all frames)
  - **[I]**$\varphi \to \varphi$ (for every formulae φ) is **T**-valid (only valid in the class of reflexive frames)
  - **[I]**$\varphi \to$ **<I>**φ (for every formulae φ) is **D**-valid (only valid in the class of serial frames)
  - **[I]**$\varphi \to$ **[I][I]**φ (for every formulae φ) is **4**-valid (only valid in the class of transitive frames)
  - **<I>**$\varphi \to$ **[I]<I>**φ (for every formulae φ) is **5**-valid (only valid in the class of Euclidean frames)

Validity in a Class of Frames: Example I

- **[I]**$(\varphi \to \psi) \to ($**[I]**$\varphi \to$ **[I]**$\psi)$ is **K**-valid:
  1 Let M be a arbitrarily chosen Kripke model and w be a arbitrary world in M.
  1.1 Assume $M, w \models$ **[I]**$(\varphi \to \psi)$ (otherwise the formula is true anyway ☺). Thus, for every world u: if $(w,u) \in R(I)$ then $M, u \models \varphi \to \psi$.
  1.1.1 If **[I]**φ is false in w, then (**[I]**$\varphi \to$ **[I]**ψ) is true in w, and the overall formula is true in w. ☺
  1.1.2 If **[I]**φ is true in w, then both **[I]**$\varphi \to \psi$ and **[I]**(φ) are true in w. Thus, in every world u accessible from w, also ψ is true, i.e., **[I]**(ψ) is true in w. Therefore, the overall formula is true in w. ☺

Validity in a Class of Frames: Example II

- **[I]**$\varphi \to \varphi$ is not **K**-valid:
  - Consider Kripke model $M = (W, R, V)$ from class **K**:
    - $W = \{w\}$
    - $R(I) = \{\}$
    - $V(p) = \{\}$
  - Check that $M, w \models$ **[I]**p and $M, w \not\models p$. Thus, $M, w \not\models$ **[I]**$p \to p$.

Satisfiability Checking

- The validity problem can be reduced to the satisfiability problem:
  - Instead of asking whether φ is true in all worlds in all Kripke models in a class, we can ask if $\neg\varphi$ is true in some world in some Kripke model in the class.
- Problem formulation:
  - Input: A formula φ.
  - Output: “Yes” if there is a Kripke model M and a world w of M such that $M, w \models \varphi$, “No” otherwise.
- It turns out that we can systematically search for Kripke models that satisfy some formula. With this tool at hand, we can algorithmically decide the satisfiability and validity problem of (most) modal logics. ⇒On Monday! ☺

Literature I

- M. Wooldridge, An Introduction to MultiAgent Systems, John Wiley & Sons, 2002.
- O. Gasquet, A. Herzig, B. Said, F. Schwarzentruber, Kripke's Worlds — An Introduction to Modal Logics via Tableaux, Springer, ISBN 978-3-7643-8503-3, 2014.