Introduction to Game Theory

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Exercise Sheet 11

Due: Monday, July 19, 2017

Exercise 11.1 (Security Games I, 0 points)

Prove: For all SSE $\langle \alpha_\mathrm{d}, g \rangle$ and all NE $\langle \alpha'_\mathrm{d}, \alpha'_\mathrm{a} \rangle$ it holds that $U_\mathrm{d}(\alpha_\mathrm{d}, g(\alpha_\mathrm{d})) \geq U_\mathrm{d}(\alpha'_\mathrm{d}, \alpha'_\mathrm{a})$.

Exercise 11.2 (Security Games II, 0 points)

Prove: Let $\mathcal{G} = \langle T, R, (S_i), U_\mathrm{d}^\mathrm{c}, U_\mathrm{d}^\mathrm{u}, U_\mathrm{a}^\mathrm{c}, U_\mathrm{a}^\mathrm{u} \rangle$ be a security game and $\overline{\mathcal{G}} = \langle T, R, (S_i), \overline{U_\mathrm{d}^\mathrm{c}}, \overline{U_\mathrm{d}^\mathrm{u}}, U_\mathrm{a}^\mathrm{c}, U_\mathrm{a}^\mathrm{u} \rangle$ be the corresponding zero-sum game with $\overline{U_\mathrm{d}^\mathrm{c}} = -U_\mathrm{a}^\mathrm{c}$ and $\overline{U_\mathrm{d}^\mathrm{u}} = -U_\mathrm{a}^\mathrm{u}$. Then $\langle \alpha_\mathrm{d}, \alpha_\mathrm{a} \rangle$ is a Nash equilibrium in $\mathcal{G}$ iff $\langle \alpha_\mathrm{d}, f(\alpha_\mathrm{a}) \rangle$ is a Nash equilibrium in $\overline{\mathcal{G}}$.

Recall: $f\colon \alpha_\mathrm{a} \mapsto \overline{\alpha_\mathrm{a}}$ is defined as $\overline{\alpha_\mathrm{a}}(t_i) = \lambda \cdot \alpha_\mathrm{a}(t_i) \cdot \frac{\Delta U_\mathrm{d}(t_i)}{\Delta U_\mathrm{a}(t_i)}$ where $\lambda > 0$ is a normalizing constant such that $\sum_{i=1}^{n} \overline{\alpha_\mathrm{a}}(t_i) = 1$.

The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.