

## Multi-Agent Systems

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### Exercise Sheet 6

Due: June 27th, 2016, 10:00

**Exercise 6.1** (Epistemic Logic and Deontic Logic, 3+5)

- (a) Show that the axioms K, T, and 5 are valid in the class of reflexive, euclidian Kripke frames.
- (b) Prove the following theorems and discuss their appropriateness in the context of modeling epistemic/normative concepts:<sup>1</sup>
- $\vdash_{S5} \neg K(\phi \wedge \neg K\phi)$
  - $\vdash_{S5_n} K_j K_i \phi \rightarrow K_j \phi$
  - $\vdash_{SDL} O(\phi \wedge \psi) \rightarrow O\psi$
  - $\not\vdash_{SDL} P(\phi \vee \psi) \rightarrow (P\phi \wedge P\psi)$
  - $\vdash_{LKA-2} O(O\phi \rightarrow \phi)$

**Exercise 6.2** (Announcement Logic, 2+1+1+1+2)

We want to model and solve a simple logics puzzle using Public Announcement Logic (PAL).<sup>2</sup> Consider the following situation with two agents. Each of them has a number written on its forehead, which can only be seen by the other agent. Also, it is common knowledge between the agents that both numbers have to be consecutive integers between 0 and 9. Furthermore, let us assume that we are Agent 1 and see the number 4 on Agent 2's forehead. Now consider the following sequence of truthful announcements:

Agent 1: "I don't know my number!"  
Agent 2: "I don't know my number!"  
Agent 1: "I don't know my number!"  
Agent 2: "I don't know my number!"  
Agent 1: "I know my number!"

- (a) Model both possible initial situations as pointed Kripke models.
- (b) Model the announcements from the dialogue as public announcement actions.
- (c) What is your number? Formulate your hypothesis as formula in PAL.
- (d) Will Agent 2 know his number? Formulate your hypothesis as formula in PAL.
- (e) Apply model checking to verify your hypotheses.

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<sup>1</sup>Recall: Epistemic Logic: **S5**, Multi-Agent Epistemic Logic: **S5<sub>n</sub>**, Standard Deontic Logic (SDL): **KD**, Leibniz-Kanger-Anderson reduction (LKA-2): **KDT**

<sup>2</sup>Different versions of this puzzle and their origins are discussed in the book *One Hundred Prisoners and a Light Bulb* by Hans van Ditmarsch and Barteld Kooi, Springer, 2015.