Multi-Agent Systems

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Summer Semester 2016
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Exercise Sheet 5

Due: June 20th, 2016, 10:00

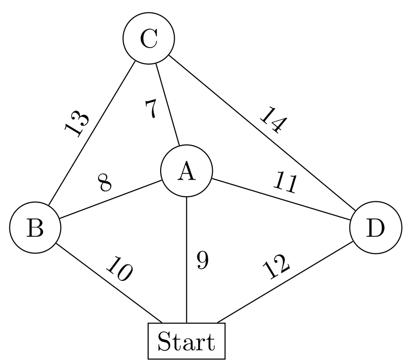
Exercise 5.1 (The Shapley Value, 3+3)

(a) Prove that the Shapley value satisfies the axioms *Symmetry*, *Dummy player*, and *Additivity*.

(b) Prove that the Shapley value is *feasible* and *efficient*.

Exercise 5.2 (Taxi Cost Sharing, 3+3+3+3+3)

Consider a scenario for sharing taxis and taxi fares that is slightly more complex than the one presented in the lecture. There are four agents who each want to be transported from a common starting point (a taxi stand) to their individual destinations (A, B, C, and D). The costs a taxi charges for a route can be calculated by adding up the costs for the single route segments, which are given as edge labels in the following graph:



E.g., the route Start-A-C has a cost of $9 + 7 = 16$. Since taxis take always the shortest and cheapest route, the taxi fare for a coalition is always the cost of the cheapest route starting from "Start" and containing all the destinations of the agents in the coalition.

(a) Model the problem as cooperative game (N, v).

(b) Calculate the Shapley value $\Psi(N, v)$.

(c) Find the socially optimal coalition structure CS^* for (N, v).

(d) Calculate the Shapley value $\Psi(N', v)$ for each coalition $N' \in CS^*$.

(e) Which of the Shapley values $\Psi(X, v)$ calculated in (b) and (d) are in the core of their respective games (X, v)? Prove your answer!