Introduction to Game Theory

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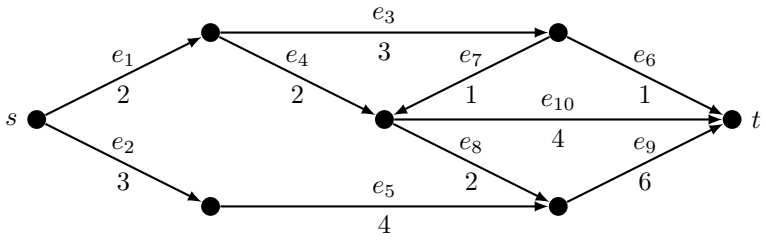
Exercise Sheet 9

Due: Thursday, July 7, 2016

Exercise 9.1 (Network routing as VCG-mechanism, 4 points)

Let $G = (V, E)$ be a directed graph. Every edge $e \in E$ belongs to a player e and generates cost c_e when being used. We want to rent a path between the two nodes s and t. The set of alternatives A contains all paths between s and t. Player e has cost c_e, if edge e lies on the chosen path p, zero otherwise. Maximization of social welfare means minimizing $\sum_{e \in p} c_e$ over all paths p from s to t.

Which alternatives does the VCG-mechanism choose in the following concrete example? Which payments result from this?
Please justify your answers.



Exercise 9.2 (VCG-mechanism and incentive compatibility, 4 points)

Let $\mathcal{M} = (f, p_1, \ldots, p_n)$ be a VCG-mechanism with Clarke-Pivot function. Then $\mathcal{M}$ is incentive compatible and has no positive transfers. Consider the mechanism $\mathcal{M}' = (f, p'_1, \ldots, p'_n)$ with

$$p'_i(v_1, \ldots, v_n) = p_i(v_1, \ldots, v_n) - \frac{1}{n} \sum_{j=1}^{n} p_j(v_1, \ldots, v_n)$$

for $i = 1, \ldots, n$.

The intuition is that $\mathcal{M}'$ chooses the same alternative as $\mathcal{M}$, the same payments are demanded from the players first, but in the end the excess money will be transferred back to the players in equal shares.

Show by counter example that $\mathcal{M}'$ is not incentive compatible.

The exercise sheets may and should be worked on and handed in in groups of two to three students. Please indicate all names on your solution.