Introduction to Modal Logic

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Exercise Sheet 10

Due: 08-07-2015

The exercise sheets may be worked on, and handed in, in groups of two students. In that case please indicate both names on your solution. The solution must be handed in on Wednesday *before* the lecture (either on paper or electronically by email to `woelfl@informatik.uni-freiburg.de`).

Exercise 10.1 (3 points)

We consider the following modal logics **KE**, **KDE**, and **KB4**. For each of these logics find a minimal set of affirmative modalities (such that each other modality can be reduced to a modality in the minimal set). Provide and briefly explain their modality diagrams, respectively.

Exercise 10.2 (3 points)

Apply the tableau procedure to decide the following questions:

(a) Is the formula $\Diamond\Box p \wedge \Box\Diamond q \rightarrow \Diamond\Diamond(p \wedge q)$ **K**-valid?

(b) Is the set of formulae

$$\{\Diamond p \vee \Diamond q,\ \Box\neg q \leftrightarrow \Box\neg p,\ \Box(p \rightarrow \neg q \wedge \Box\neg q),\ \Box(q \rightarrow \neg p \wedge \Box\neg p)\}$$

K-satisfiable?

(c) In system **KT**, does $\Box\Box(p \wedge q)$ entail $\Box\Diamond(q \wedge \Box\Diamond p)$?

Exercise 10.3 (4 points)

Consider the following axiomatization for regular PDL, with axioms:

(PDL1) $[\pi](p \rightarrow q) \rightarrow ([\pi]p \rightarrow [\pi]q)$

(PDL2) $\langle\pi\rangle p \leftrightarrow \neg[\pi]\neg p$

(PDL3) $\langle\pi_1;\pi_2\rangle p \leftrightarrow \langle\pi_1\rangle\langle\pi_2\rangle p$

(PDL4) $\langle\pi_1 \cup \pi_2\rangle p \leftrightarrow \langle\pi_1\rangle p \vee \langle\pi_2\rangle p$

(PDL5) $\langle\pi^*\rangle p \leftrightarrow (p \vee \langle\pi\rangle\langle\pi^*\rangle p)$

(PDL6) $p \wedge [\pi^*](p \rightarrow [\pi]p) \rightarrow [\pi^*]p$

and rules: modus ponens, uniform substitution, and necessitation ($\vdash \varphi \implies \vdash [\pi]\varphi$).

(a) Show that the axiomatization is sound with respect to the class of regular PDL frames (i.e., relational frames that satisfy the regularity conditions listed in Example 2.13). It suffices to consider (PDL5) and (PDL6).

(b) Is this axiomatization strongly complete with respect to the class of all regular PDL frames?

Hint: Consider the set of formulae

$$\{p, [a]p, [a^2]p, [a^3]p, \dots\} \cup \{\neg[a^*]p\}$$

where $a^k := a$ for $k = 1$ and $a^k := (a; a^{k-1})$ for $k > 1$. Is this set inconsistent?