Introduction to Modal Logic

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Exercise Sheet 9

Due: 01-07-2015

The exercise sheets may be worked on, and handed in, in groups of two students. In that case please indicate both names on your solution. The solution must be handed in on Wednesday *before* the lecture (either on paper or electronically by email to `woelfl@informatik.uni-freiburg.de`).

Exercise 9.1 (4 points)

Let Λ be a normal modal logic, S^Λ the set of all maximal Λ-consistent sets of $\mathcal{L}(P)$-formulae, and $\mathcal{F}^\Lambda = \left\langle S^\Lambda, \{R^\Lambda_\Diamond\}_{\Diamond\in\tau}\right\rangle$ the canonical frame of Λ. Show:

(a) If $\mathbf{D} \in \Lambda$, then the canonical relation $R^\Lambda_\Diamond$ is serial.

(b) If $\mathbf{4} \in \Lambda$, then $R^\Lambda_\Diamond$ is transitive.

(c) If $\mathbf{E} \in \Lambda$, then $R^\Lambda_\Diamond$ is Euclidean.

Exercise 9.2 (6 points)

Consider the frames $\mathcal{F}_0 = \langle\{0\}, \emptyset\rangle$ and $\mathcal{F}_1 = \langle\{1\}, \{(1,1)\}\rangle$ as well as their induced normal modal logics $\Lambda_0 := \Lambda(\mathcal{F}_0)$ and $\Lambda_1 := \Lambda(\mathcal{F}_1)$.

(a) Provide axiomatizations of Λ_0 and Λ_1 (show soundness and completeness).

(b) Which of these modal logics is/are canonical?

(c) Show that each canonical normal modal logic is contained (as subset) in Λ_0 or in Λ_1.

(d) In which of the modal logics Λ_0 or Λ_1 are the logics **S4**, **KBE**, and **KL** contained?

(e) Show that there is exactly one normal modal logic that contains both Λ_0 and Λ_1.