Introduction to Modal Logic

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Exercise Sheet 8

Due: 24-06-2015

The exercise sheets may be worked on, and handed in, in groups of two students. In that case please indicate both names on your solution. The solution must be handed in on Wednesday *before* the lecture (either on paper or electronically by email to `woelfl@informatik.uni-freiburg.de`).

In the following exercises we consider a basic modal language $\mathcal{L}$ with a single diamond.

Exercise 8.1 (3 points)

Consider the formula:

Alt$_n$ $\Box p_1 \vee \Box(p_1 \to p_2) \vee \cdots \vee \Box(p_1 \wedge \cdots \wedge p_n \to p_{n+1})$

Show that for $n \in \mathbb{N}$ this formula defines the class of Kripke frames in which each state sees at most n successor states.

Exercise 8.2 (3 points)

Prove the following theorems and derived rules:

(a) $\vdash_{\mathbf{K}} (\Diamond\varphi \wedge \Box\psi) \to \Diamond(\varphi \wedge \psi)$

(b) $\vdash_{\mathbf{KTE}} \Diamond\Diamond\varphi \to \Diamond\varphi$

(c) If $\vdash_{\mathbf{KB}} \Diamond\varphi \to \psi$, then $\vdash_{\mathbf{KB}} \varphi \to \Box\psi$.

Exercise 8.3 (4 points)

Let Λ be a modal logic and Σ be a maximal Λ-consistent set of formuale. Show the following claims (for the proof you may use only (a) and (b) of Prop. 3.10 in the lecture notes):

(a) For each $\mathscr{L}_\tau(P)$-formula φ, $\varphi \in \Sigma$ or $\neg\varphi \in \Sigma$.

(b) For each $\mathscr{L}_\tau(P)$-formula φ, $\neg\varphi \in \Sigma$ if and only if $\varphi \notin \Sigma$.

(c) For $\mathscr{L}_\tau(P)$-formula φ and ψ, $\varphi \wedge \psi \in \Sigma$ if and only if $\varphi \in \Sigma$ and $\psi \in \Sigma$.

(d) For each $\mathscr{L}_\tau(P)$-formula φ, if $\Sigma \vdash_\Lambda \varphi$, then $\varphi \in \Sigma$.