Introduction to Modal Logic

B. Nebel, S. Wölfl | University of Freiburg
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Exercise Sheet 7

Due: 17-06-2015

The exercise sheets may be worked on, and handed in, in groups of two students. In that case please indicate both names on your solution. The solution must be handed in on Wednesday *before* the lecture (either on paper or electronically by email to `woelfl@informatik.uni-freiburg.de`).

In the following exercises we consider a basic modal language $\mathcal{L}$ with a single modal operator.

Exercise 7.1 (3 points)

a) Show that for each Kripke frame $\mathcal{F}$, the set of formulae

$$\Lambda(\mathcal{F}) = \{\varphi : \mathcal{F} \models \varphi\}$$

is a normal modal logic.

b) Show that a similar claim for Kripke models does not hold in general: provide a model $\mathcal{M}$ such that

$$\Lambda(\mathcal{M}) = \{\varphi : \mathcal{M} \models \varphi\}$$

is not a normal modal logic.

Exercise 7.2 (2 points)

Show that the modal logic **KTL** is the inconsistent modal logic.

Exercise 7.3 (2 points)

Show that the axiom $\mathbf{G1} = \Diamond\Box p \to \Box\Diamond p$ defines the class of all frames $\mathcal{F}$ with

$$\mathcal{F} \models \forall x y_1 y_2 (xRy_1 \wedge xRy_2 \to \exists z (y_1 Rz \wedge y_2 Rz)).$$

Exercise 7.4 (3 points)

Show that **KTE** = **KT4B**.

Hint: It suffices to show E $\in$ **KT4B** and 4, B $\in$ **KTE**. For the second part of the proof it could be useful to show first that B $\in$ **KTE**.