Introduction to Modal Logic

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Exercise Sheet 6

Due: 10-06-2015

The exercise sheets may be worked on, and handed in, in groups of two students. In that case please indicate both names on your solution. The solution must be handed in on Wednesday *before* the lecture (either on paper or electronically by email to `woelfl@informatik.uni-freiburg.de`).

Exercise 6.1 (3 points)

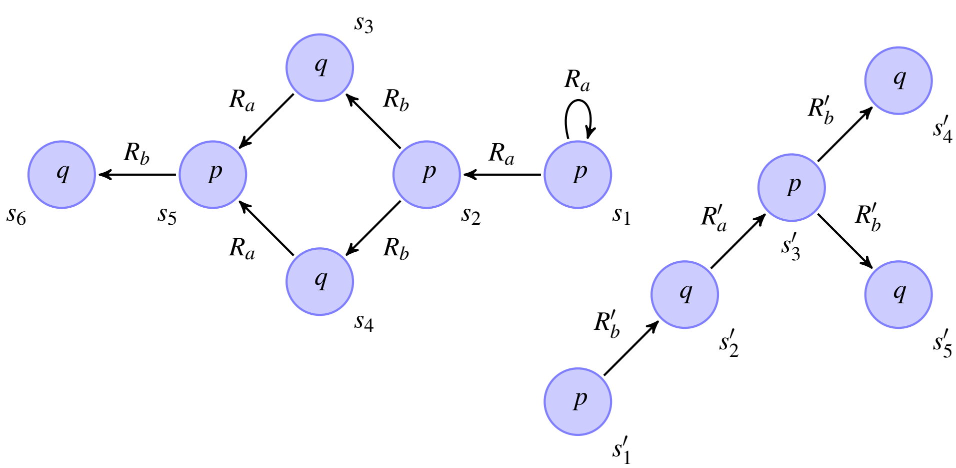
We consider the basic modal language $\mathcal{L}$ with a single modal operator $\Diamond$. Show the following claims:

(a) Each satisfiable formula is satisfiable in tree-like model (a tree-like model here is a Kripke model with a designated root state such that each state is reachable from the root by exactly one R-chain).

(b) Each satisfiable formula is satisfiable on an irreflexive, intransitive, and asymmetric frame.

Exercise 6.2 (4 points)

Consider the following two models $\mathcal{M} = \langle S, R_a, R_b, V \rangle$ and $\mathcal{M}' = \langle S', R'_a, R'_b, V' \rangle$ for a basic modal language with $P = \{p, q\}$.



(a) Show that none of the models is the image of the other model under some bounded morphism.

(b) Are the models bisimilar, i.e., does there exist a bisimulation between the models? If there exists a bisimulation, provide a minimal and a maximal one (minimal and maximal bisimulations are defined in terms of set inclusion, e.g., a bisimulation Z is maximal if there exists no bisimulation Z' with $Z \subsetneq Z'$).

Exercise 6.3 (3 points)

The proof of the Hennessy-Milner theorem uses the assumption that the models are image-finite. In fact, this assumption is crucial. Show this by defining two models $\mathcal{M}$ and $\mathcal{M}'$ with states s and s' such that $s \equiv_{\mathcal{M},\mathcal{M}'} s'$, but not $s \leftrightsquigarrow_{\mathcal{M},\mathcal{M}'} s'$.