Introduction to Modal Logic

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Exercise Sheet 4

Due: 20-05-2015

The exercise sheets may be worked on, and handed in, in groups of two students. In that case please indicate both names on your solution. The solution must be handed in on Wednesday *before* the lecture (either on paper or electronically by email to `woelfl@informatik.uni-freiburg.de`).

Exercise 4.1 (3 points)

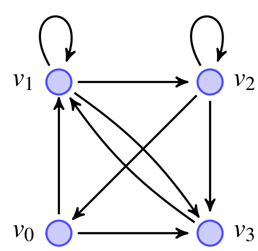
Consider the *difference operator* $\mathsf{D}\varphi$ semantically defined by

$$\mathcal{M} \models_s \mathsf{D}\varphi \iff \text{there exists a state } s' \neq s \text{ such that } \mathcal{M} \models_{s'} \varphi$$

Is the operator definable in the basic modal language (unary diamonds only)?

Exercise 4.2 (4 points)

Consider the following directed graph G.



Let $\mathcal{F} = \langle A, C, R, I \rangle$ be the arrow frame defined by this graph (where A is the set of arcs in G).

a) Determine the relations C (composition), R (reverse), and I (skip) from the graph.

b) Which of the following formulae are valid on that frame, which are not? Provide proofs or counter-examples.

(1) $1' \circ \otimes p \leftrightarrow \otimes p$

(2) $\otimes(p \circ q) \leftrightarrow (\otimes q \circ \otimes p)$

(3) $((p \circ q) \circ r) \leftrightarrow (p \circ (q \circ r))$.

Exercise 4.3 (3 points)

We consider two frames: $\mathcal{N} = \langle \mathbb{N}, < \rangle$ is the frame of the natural numbers together with their natural ordering. The frame $\mathcal{B} = \langle S, R \rangle$ is defined as follows: its states are the words over the alphabet $\{0, 1\}$ (i.e., all finite strings of 0s and 1s); the accessibility relation is defined by wRw' iff the word w is a proper inital substring of w'.

(a) Let V be the valuation on $\mathcal{N}$ defined by: $V(p) := \{2n : n \in \mathbb{N}\}$ (for each propositional variable p). Define a valuation V' on $\mathcal{B}$ and a mapping $f : S \to \mathbb{N}$ such that f is a bounded morphism from $\langle \mathcal{B}, V' \rangle$ to $\langle \mathcal{N}, V \rangle$.

(b) CORRECTED:

Let V' be the valuation on $\mathcal{B}$ given by: $V'(p) := \{1w : w \in S\}$ (for each propositional variable p). Show that there exists no valuation V on $\mathcal{N}$ such that one can define a bounded morphism f from $\langle \mathcal{B}, V' \rangle$ to $\langle \mathcal{N}, V \rangle$.