Introduction to Modal Logic

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Exercise Sheet 3

Due: 13-05-2015

The exercise sheets may be worked on, and handed in, in groups of two students. In that case please indicate both names on your solution. The solution must be handed in on Wednesday *before* the lecture (either on paper or electronically by email to `woelfl@informatik.uni-freiburg.de`).

Exercise 3.1 (2 points)
Explain how the following programming statements can be expressed in PDL:

(a) `while` φ `do` π

(b) `repeat` π `until` φ

Exercise 3.2 (4 points)
Consider the following PDL-formulae. Which of them are valid in each PDL model, which are not. Provide proofs or counter-examples.

(a) $\langle \pi_1 ; \pi_2 \rangle\, p \leftrightarrow \langle \pi_1 \rangle\, [\pi_2]\, p$

(b) $[\pi_1 \cup \pi_2] p \leftrightarrow ([\pi_1] p \wedge [\pi_2] p)$

(c) $\langle q? \rangle\, p \leftrightarrow (q \wedge p)$

(d) $[\pi][\pi^*] p \rightarrow [\pi^*] p$

Recall: $R_{\pi_1;\pi_2} = R_{\pi_1} \circ R_{\pi_2}$, $R_{\pi_1 \cup \pi_2} = R_{\pi_1} \cup R_{\pi_2}$, $R_{\pi^*} = R^*_\pi$, and $R_{\varphi?} = \{(s,s') \in S \times S \colon s = s' \text{ and } s \models \varphi\}$.

Exercise 3.3 (4 points)
Show how LTL can be embedded into PDL. More exactly, given an LTL-language $\mathcal{L} = \mathcal{L}(P)$ with modal connectives F, X and U, provide

(a) a PDL-language $\mathcal{L}'$ (propositional variables, atomic programs, PDL constructs);

(b) the recursive definition of a function Φ that assigns to each $\mathcal{L}$-formula φ a $\mathcal{L}'$-formula $\Phi(\varphi)$; and

(c) a function that assigns to each LTL model, i.e., each infinite sequence $s = s_1\, s_2 \ldots$ of truth assignments $s_i \colon P \rightarrow \{0,1\}$, a PDL model $\mathcal{M}^s$

such that

$$s \models \varphi \iff \mathcal{M}^s \models_{s_1} \Phi(\varphi). \tag{1}$$

Hint: For the proof of the equivalence (1) show the stronger claim $s^i \models \varphi \Leftrightarrow \mathcal{M}^s \models_{s_i} \Phi(\varphi)$ for $i \in \mathbb{N}$.