Introduction to Modal Logic

B. Nebel, S. Wölfl
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University of Freiburg
Department of Computer Science

Exercise Sheet 1

Due: 29-04-2015

The exercise sheets may be worked on, and handed in, in groups of two students. In that case please indicate both names on your solution. The solution must be handed in on Wednesday *before* the lecture (either on paper or electronically by email to `woelfl@informatik.uni-freiburg.de`).

Exercise 1.1 (3 points)

Prove Lemma 1.2 (Substitution Lemma).

Exercise 1.2 (3 points)

Complete the proof sketch of Lemma 1.3 (Interpolation Theorem) given in the lecture.

Exercise 1.3 (4 points)

Consider the following modal logic formulae:

(1) $\Box(\varphi \wedge \psi) \leftrightarrow (\Box\varphi \wedge \Box\psi)$

(2) $\Diamond(\varphi \wedge \psi) \leftrightarrow (\Diamond\varphi \wedge \Diamond\psi)$

(3) $\Diamond\Box\varphi \rightarrow \Box\varphi$

(4) $\Diamond\varphi \rightarrow \Diamond\Diamond\varphi$

Discuss for each formula whether it is "logically" valid under the following readings of the box operator:

(a) $\Box\varphi$: "the agent believes that φ is true"

(b) $\Box\varphi$: "it has always been the case that φ is true"

Remark. If the reading is ambiguous, explain how you understand it. Moreover, $\Diamond\varphi$ is to be read as $\neg\Box\neg\varphi$.